Hokkaido Mathematical Journal Vol. 13 (1984) p. 133-144

Kernels associated with cylindrical measures on locally convex spaces

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(Received August 8, 1983)

§ 1. Introduction

The present paper contains some results concerning kernels of cylindrical measures on locally convex spaces. The notion of kernel has been introduced by C. Borell [3].

Let E be a locally convex space, E^{*} be its topological dual space, μ be a cylindrical measure on E and $L: E^{*}\rightarrow L^{0}(\Omega, \Sigma, P)$ be a random linear functional associated with μ. The inverse image of the topology of the convergence in probability on $L^{0}(\Omega, \Sigma, P)$ under L is called the topology associated with μ and denoted by τ_{μ}; τ_{μ} is a linear topology on E^{*}. The topological dual of (E^{*}, τ_{μ}) is called the kernel of μ and denoted by K_{μ}. Let τ be a linear topology on E^{*}. The cylindrical measure μ is called of type 0 with respect to τ if the random linear functional $L: (E^{*}, \tau)\rightarrow L^{0}(\Omega, \Sigma, P)$ is continuous, and μ is called of type p (for $p>0$) with respect to τ if the image of E^{*} under L is contained in $L^{p}(\Omega, \Sigma, P)$ and $L: (E^{*}, \tau)\rightarrow L^{p}(\Omega, \Sigma, P)$ is continuous. Then our main results are stated as follows.

Let E and F be locally convex spaces, T be a continuous linear mapping of F into E, τ be a linear topology on E^{*} and τ_{k} be the Mackey topology on F^{*}. Then it is shown that the adjoint mapping $T^{*}: (E^{*}, \tau)\rightarrow(F^{*}, \tau_{k})$ can be factored through a subspace of $L^{0}(\Omega, \nu)$ for some probability space (Ω, ν) if and only if there exists a cylindrical measure μ on E of type 0 with respect to τ such that K_{μ} contains $T(F)$. In this case, if F is quasi-complete or barrelled, then τ_{k} can be replaced by the strong topology $b(F^{*}, F)$. As a special case, we can give a characterization of L^{0}-imbeddable spaces, which is similar to the results of S. Chevet [4] and Y. Okazaki [8]. For $p>0$, it is also shown that if there exists a cylindrical measure μ on E of type p with respect to τ such that K_{μ} contains $T(F)$, then $T^{*}: (E^{*}, \tau)\rightarrow(F^{*}, \tau_{k})$ can be factored through a subspace of $L^{p}(\Omega, \nu)$ for some probability space (Ω, ν). In this case, if $p=2$, then the converse is also true. Here we are

The research by this author was partially supported by Grant-in-Aid for Scientific Research (No. 57540086), Ministry of Education.