The weak Behrens' property and the corona

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Abstract: In this paper we study a class of infinitely connected domains larger than the one considered by Behrens [1] and prove that the corona problem has an affirmative answer.

Introduction. Let D be a bounded domain in the complex plane and $H^\infty(D)$ be the algebra of bounded analytic functions on D. The corona problem asks whether D is weak* dense in the space $\mathscr{M}(D)$ of maximal ideals of $H^\infty(D)$. Carleson [3] proved that the open unit disc $\varDelta$ is dense in $\mathscr{M}(\varDelta)$. In [7] Stout extended Carleson's result to finitely connected domains. In [1] Behrens found a class of infinitely connected domains for which the corona problem has an affirmative answer. In this paper we will use Behrens' idea to extend the results to more general domains. See [4] and [5] for other extensions.

By a $\varDelta$-domain we mean a domain D obtained from the open unit disc $\varDelta$ by deleting the origin and a sequence of disjoint closed discs $\varDelta_n = \varDelta(c_n, r_n) = \{z \in C : |z - c_n| \leq r_n\}$ with $c_n \to 0$. Under the additional hypothesis $\sum \frac{r_n}{|c_n|} < \infty$, Zalcman showed in [8] that there is a distinguished homomorphism in $\mathscr{M}(D)$ defined by

$$\varphi_0(f) = \frac{1}{2\pi i} \int_{\partial D} \frac{f(z)}{z} dz.$$

The distinguished homomorphism φ_0, if it exists, is always adherent to D [6]. Behrens showed that if there are numbers $R_n > r_n$ such that $\sum \frac{r_n}{R_n} < \infty$ and the discs $D_n = \varDelta(c_n, R_n)$ are disjoint, then D is dense in $\mathscr{M}(D)$. Such a domain is called a Behrens' domain.

Notations and terminology: Throughout we assume that D is a $\varDelta$-domain and that there exists numbers $R_n > r_n$ such that the discs $D_n = \varDelta(c_n, R_n)$ are disjoint and $\frac{r_n}{R_n} \to 0$. Let $E_n = \frac{r_n}{z - c_n}$ for $z \in \varDelta_n^c = C \backslash \varDelta_n$, $n = 1, 2, \cdots$. Let $s_n = \sqrt{r_n R_n}$, so $\frac{r_n}{s_n} \to 0$ and $\frac{s_n}{R_n} \to 0$, and let $B_n = \varDelta(c_n, s_n)$. Let $H^\infty(\varDelta \times N)$ be the algebra of bounded functions which are analytic on each slice of $\varDelta \times N$