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Lifting modules, extending modules and their applications to generalized uniserial rings

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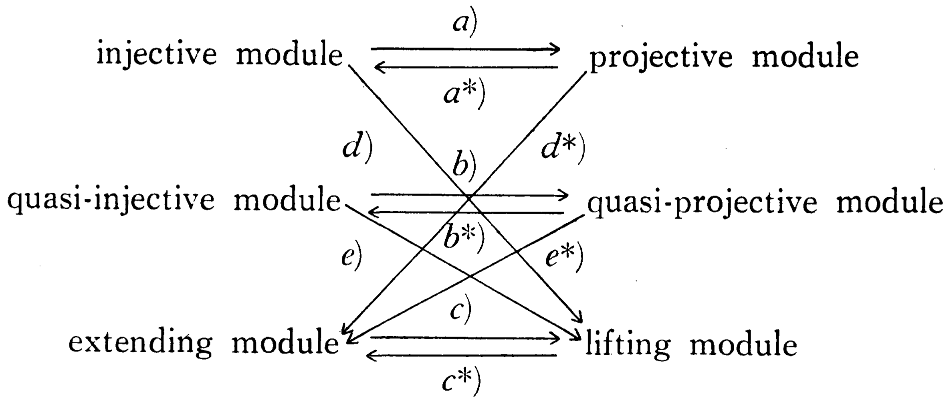
Artinian serial principal ideal rings and artinian serial rings are traditionally called uniserial rings and generalized uniserial rings, respectively. These rings are important classical artinian rings as well as quasi-Frobenius rings. The reader is referred to Faith's Book [4] for these rings. As is well known, a ring R is a quasi-Frobenius ring iff every injective R-module is projective, and if every projective R-module is injective; while R is a uniserial ring iff every quasi-injective R-module is quasi-projective, and iff every quasi-projective R-module is quasi-injective ([2], [3], [5]).

The purpose of this paper is to give similar characterizations of a generalized uniserial ring R in terms of extending and lifting modules. More specifically, consider the following implications:



As just noted above, R is quasi-Frobenius$\Leftrightarrow a)\Leftrightarrow a^{*})$; while R is uniserial$\Leftrightarrow b)$ $\Leftrightarrow b^{*})$. The conditions $d)$ and $d^{*})$ are recently studied by Harada ([6]$\sim$[8]) and Oshiro ([15]). In this paper, we study $c)$, $c^{*})$, $e)$ and $e^{*})$ and show the following result: R is generalized uniserial$\Leftrightarrow e)\Leftrightarrow e^{*})\Leftrightarrow c)\Leftrightarrow R$ is a right perfect ring with $c^{*})$.

NOTATION. Throughout this paper, we assume that R is an associative ring with identity and all R-modules are unitary right R-modules. Let M be an R-module. We use $E(M)$, $J(M)$ $Soc(M)$ to denote the injective