Hokkaido Mathematical Journal, Vol. 13 (1984) p. 299-309

Characterization of Stieltjes transforms of vector measures and an application to spectral theory

By Werner RICKER

(Received June 4, 1983)

Abstract

The classical result of D. V. Widder characterizing those complex-valued functions on $(0, \infty)$ which are the Stieltjes transform of a complex measure on $[0, \infty)$, is generalized to functions with values in a quasi-complete locally convex space. This result is then used to establish a criterion for operators with spectrum in $[0, \infty)$ to be scalar-type spectral operators.

Introduction

Let M and D respectively denote the formal operators of multiplication $M: f(t)\mapsto tf(t)$ and differentiation $D: f\rightarrow f'$. The (formal) Widder differential operators L_k are given by

$$L_k = c_k M^{k-1} D^{2k-1} M^k, \qquad k=1, 2, \cdots, \tag{1}$$

where $c_1=1$ and $c_k=(-1)^k[k!(k-2)!]^{-1}$ for $k\geq 2$.

It is known that a complex-valued function f on $(0, \infty)$ can be characterized as a Stieltjes transform in terms of the maps $L_k(f)$, $k=1, 2, \cdots$. Namely, there exists a (unique) regular complex Borel measure m on $[0, \infty)$ such that

$$f(t) = \hat{m}(t) = \int_0^\infty (s+t)^{-1} dm(s), \qquad t\in(0, \infty), \tag{2}$$

if and only if f has derivatives of all orders in $(0, \infty)$ and there exists a constant K such that

$$\int_0^\infty \left| L_k(f)(t) \right| dt \leq K, \qquad k=1, 2, \cdots, \tag{3}$$

(see [8], VIII Theorem 16 or [4], p. 165).

Let C_0 denote the space of all continuous complex-valued functions on $[0, \infty)$ which vanish at infinity, equipped with the uniform norm. Then condition (3) means that the maps $\Phi_k(f)$, $k=1, 2, \cdots$, defined by