Hokkaido Mathematical Journal, Vol. 13 (1984) p. 285-298

Corestriction and p-subgroups

By Tadashi IKEDA

(Received February 7, 1984)

1. Introduction

Let G be a finite group, p a prime number. In his study on the modular representation, J. A. Green defined G-algebras and defect groups for G-algebras [6]. After his ideas, M. Broué and L. Puig defined the Brauer homomorphism for G-algebras and obtained the "First Fundamental Theorem" for G-algebras. The classical "First Fundamental Theorem" for blocks and the Green correspondence was shown as corollaries of this theorem. They also gave the definitions of interior G-algebras and the "corestriction" of interior G-algebras as ring theoretical version of the induction of modules and showed the extention of Higman's criterion for relative projectivity and Green's theorem on the induction of absolutely indecomposable modules. See [1], [2] and [6].

On the other hand, in his paper [3], D. W. Burry showed the relation between blocks and induced modules from p-subgroups. Our purpose in this paper is to extend the Burry's result to interior G-algebras. We shall prove Theorem 1, 2, and Corollary 3.

THEOREM 1. *Let P be a p-subgroup of G, (B, σ) a local interior P-algebra with with defect group P and $(Cor_p{}^G B, \sigma^G)$ the corestriction of (B, σ). Let Br_P be the Brauer homomorphism of $(Cor_P{}^G B, \sigma^G)$ with respect to P. If b is a block of RG whose defect group contains P up to G-conjugacy, then the element $Br_P \circ \sigma^G$ (b) is non-zero.*

THEOREM 2. *Let b be a block of RG and P a subgroup of a defect group of b. Then for a local interior P-algebra (B, σ) with defect group P there exists a local interior G-algebra (A, ρ) satisfying the following conditions:*

(1) *The element $\rho(b)$ is non-zero.*

(2) *A defect group of (A, ρ) equals P up to G-conjugacy.*

(3) *The interior P-algebra (B, σ) is a source of the interior G-algebra (A, ρ).*

COROLLARY 3. (*Burry*) *Let b be a block of RG and P a subgroup*