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Non-existence of higher order non-singular holomorphic immersions

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0. Introduction

In [6] Pohl formulated and studied the higher order complex analytic geometry and recently in [10] Watanabe studied higher order non-singular holomorphic embeddings of algebraic manifolds into Grassmann manifolds. In this note we study non-existences of higher order non-singular holomorphic immersions of complex projective spaces and their non-singular complex hypersurfaces into complex projective spaces by means of Chern classes. Our main results are Theorem 2. 2 and Corollary 3. 3. It is well known that non-singular complex algebraic curves of degree >2 in a complex projective plane have inflection points. The statement (iii) of Corollary 3. 3 is a generalization of this fact to a case of higher dimension and higher order. Let P_m be the m-dimensional complex projective space and for $q\geq 2$, we denote a non-singular complex hypersurface of degree q in P_{n+1} by $V_n(q)$. In [2] Feder proved the following theorem.

THEOREM 0. 1. *If f: $P_n\rightarrow P_N$ is a holomorphic immersion and $N<2n$, then* $\deg(f)=1$, *where* $\deg(f)$ *is a degree of f* (see Section 2 of this note).

Furthermore in [7] Samsky proved the following theorem.

THEOREM 0. 2. *If f: $V_n(q)\rightarrow P_N$ is a holomorphic immersion and $N<2n$, then* $\deg(f)=1$, *where* $\deg(f)$ *is a degree of f* (see Section 3 of this note).

In our terminology, holomorphic immersions may be regarded as first order non-singular holomorphic mappings or holomorphic mappings without 0-th order inflection points (see Section 1 of this note). Hence the statement (i) of Theorem 2. 2 (Corollary 3. 3 resp.) is a result for the higher order case of the above Theorem 0. 1 (0. 2 resp.). The proofs much depend upon symmetric power operations in K-theory which Suzuki [8, 9] firstly used in KO-theory to show non-existences of higher order non-singular differentiable immersions of real (and complex resp.) projective spaces into euclidean or real (and complex resp.) projective spaces. The author is grateful to Mr. Watanabe for enlightening conversations and advices.