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Compact submanifolds of codimension p of a Sasakian space form.

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Introduction.

In [5] M. Morohashi has shown that an n-dimensional Euclidean sphere S^{n} admits a conformal Killing tensor field of degree p for any positive integer p such that $p<n$. Then H. Kôjyô [3] has constructed a conformal Killing tensor field of degree p inductively, on a Riemannian manifold of constant curvature which admits a conformal Killing vector field. In connection with these conformal Killing tensors of degree p, they and others [1, 6, 7, 9, 13, 15] have studied submanifolds of codimension p of a sphere or a Riemannian manifold of constant curvature and have proved that these submaifolds are totally umbilical under certain conditions.

In this paper, at first we point out that there naturally exist a conformal Killing tensor field of even degree and a Killing tensor field of odd degree on a Sasakian manifold (cf. [14]). Making use of these tensors, we prove that Theorem 5.1 which gives a sufficient condition for a compact submanifold of codimension p in a Sasakian space form to be totally umbilical.

§1. Sasakian space forms.

Let M be a $(2n+1)$-dimensional manifold endowed with an almost contact metric structure (Φ, ξ, η, G), where G is a Riemannian metric, η a 1-form, ξ a vector field and Φ a tensor field of type (1.1) on M which satisfy

(1.1) $$\Phi_{\lambda}{}^{\kappa}\xi^{\lambda}=0,\ \Phi_{\lambda}{}^{\kappa}\eta_{\kappa}=0,\ \xi^{\lambda}\eta_{\lambda}=1.$$
$$\Phi_{\lambda}{}^{\kappa}\Phi_{\kappa}{}^{\nu}=-\delta_{\lambda}^{\nu}+\eta_{\lambda}\xi^{\nu},\ G_{\lambda\kappa}\Phi_{\mu}{}^{\lambda}\Phi_{\nu}{}^{\kappa}=G_{\mu\nu}-\eta_{\mu}\eta_{\nu}.$$

If, in an almost contact metric manifold M, the structure tensors (Φ, ξ, η, G) satisfy

(1.2) $$\nabla_{\mu}\Phi_{\lambda\kappa}=\eta_{\lambda}G_{\mu\kappa}-\eta_{\kappa}G_{\mu\lambda},\ \nabla_{\lambda}\xi^{\kappa}=\Phi_{\lambda}{}^{\kappa},$$

where ∇ denotes the covariant derivative with respect to the Riemannian metric $G_{\lambda\kappa}$, the structure is called a Sasakian structure and the manifold M is called a Sasakian manifold (cf. [10]). Moreover, if a Sasakian manifold