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DISAPPEARING SOLUTIONS FOR DISSIPATIVE HYPERBOLIC SYSTEMS OF CONSTANT MULTIPLICITY

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1. Introduction

Let $n\geq 3$ and Ω be an open domain in $\boldsymbol{R}^n$ with a bounded complement and boundary $\partial\Omega$ assumed real analytic and connected. Consider the mixed problem

$$(1.1)\qquad \begin{cases} (\partial_t-\sum_{j=1}^{n} A_j\partial_{x_j})u=0 & \text{on } (0,\infty)\times\Omega, \\ \Lambda(x)u=0 & \text{on } (0,\infty)\times\partial\Omega, \\ u(0,x)=f(x). \end{cases}$$

where A_j, $\Lambda(x)$ are $(r\times r)$ matrices, $\Lambda(x)$ is real analytic and $f(x)\in L^2(\Omega\,;\boldsymbol{C}^r)$. We shall assume the following conditions fulfilled

(H_1) A_j are constant Hermittian matrices,

(H_2) $\begin{cases}\text{the eigenvalues of the matrix } A(\xi)=\sum_{j=1}^{n}A_j\xi_j \\ \text{have constant multiplicity for } \xi\in\boldsymbol{R}^n\backslash\{0\}.\end{cases}$

The above conditions show that the dimension q of the positive eigenspace of the matrix $A(\xi)$ is equal to the dimension of the negative eigenspace. The boundary condition will be assumed maximal dissipative one, i. e.

(H_3) $\begin{cases}\text{a) } \langle A(\nu(x))u, u\rangle\leq 0 \text{ for } u\in\operatorname{Ker}\Lambda(x),\ x\in\partial\Omega, \\ \text{b) } \operatorname{Ker}\Lambda(x) \text{ is the maximal subspace in } \boldsymbol{C}^r, \\ \quad\ \text{satisfying the condition a).}\end{cases}$

Here $\nu(x)$ is the unit normal at $x\in\partial\Omega$ pointed into $K=\boldsymbol{R}^n\backslash\Omega$, $\langle\ ,\ \rangle$ is the inner product in $\boldsymbol{C}^r$. Moreover, we shall assume the boundary condition coercive (see [5]-[7], [18] for the precise definition). It is well known (see [12], [15], [18]) that the above conditions are valid for a wide class important physical problems such as the Maxwell's equations, accoustic wave equation, Pauli, Dirac's equations etc.

In this work we study the disappearing solutions (D. S.) to the problem