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KMO-Langevin Equation and Fluctuation-Dissipation Theorem (I)

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§ 1. Introduction

In the course of theoretical and experimental investigations for confirming the **Alder-Wainwright effect** ([1], [2], [12] and [23]), it has become clear that Brownian motion with such an effect can be described by an equation treated in G. Stokes and J. Boussinesq ([24], [3], [4] and [13]), whose equation with a random force—**Stokes-Boussinesq-Langevin equation**—gives a precise description of the time evolution of Ornstein-Uhlenbeck's Brownian motion treated in A. Einstein and P. Langevin ([5], [14] and [25]).

In [22] we have then introduced two kinds of random forces for Stokes-Boussinesq-Langevin equation : one is a white noise and the other is a Kubo noise (cf. (8. 13) in [22]) (A precise definition will be given in § 6 of this paper). And we have proved that the unique stationary solution for Stokes-Boussinesq-Langevin equation with a white noise or a Kubo noise as a random force has a qualitative nature of T-positivity and then satisfies an Alder-Wainwright effect—a long-time behavior ($\propto t^{-\frac{3}{2}}$) of velocity autocorrelation function. Next, as a generalization of Stokes-Boussinesq-Langevin equation, we have derived two kinds of Langevin equations—a first KMO-Langevin equation with a white noise as a random force and a second KMO-Langevin equation with a Kubo noise as a random force—which describe the time evolution of a real stationary Gaussian process with a qualitative nature of T-positivity. Furthermore we have clarified a mathematical stucture of the Kubo's fluctuation-dissipation theorem in his linear response theory in statistical physics ([8], [9], [10] and [11]), by proving a new type of fluctuation-dissipation theorem for the first KMO-Langevin equation and the Kubo's fluctuation-dissipation theorem for the second KMO-Langevin equation.

According to the so-called fluctuation-dissipation theorem in statistical physics, we know ([6]) that, in a physical linear system taking a reciprocal action with a microscopic and kinetic quantity which is itself doing a thermal