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On j-algebras and homogeneous Kähler manifolds

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Introduction.

The notion of j-algebras introduced by Pyatetskii-Shapiro played an important role in the theory of realization of homogeneous bounded domains as homogeneous Siegel domains. Vinberg, Gindikin and Pyatetskii-Shapiro [16] stated that the Lie algebra of a transitive holomorphic transformation group of a homogeneous bounded domain admits a structure of an effective proper j-algebra and that every effective proper j-algebra can be regarded as the Lie algebra of a transitive holomorphic transformation group of a homogeneous Siegel domain of the second kind. In this paper, we remove the properness and study the structure of homogeneous complex manifolds corresponding to effective j-algebras.

By an effective j-algebra $(\mathfrak{g}, \mathfrak{k}, j, \omega)$ we mean a system of a Lie algebra $\mathfrak{g}$, a subalgebra $\mathfrak{k}$, an endomorphism j, and a linear form ω satisfying certain conditions. (For a precise definition, see § 3.) Let G be a connected Lie group with $\mathfrak{g}$ as its Lie algebra and let K be the connected subgroup corresponding to $\mathfrak{k}$. Then K is closed and G/K admits a G-invariant Kähler structure. The homogeneous space G/K is said to be the homogeneous complex manifold associated with the effective j-algebra $(\mathfrak{g}, \mathfrak{k}, j, \omega)$. We shall prove the following theorems.

THEOREM A. *Let G/K be the homogeneous complex manifold associated with an effective j-algebra $(\mathfrak{g}, \mathfrak{k}, j, \omega)$. Then G/K is biholomorphic to a product of a homogeneous bounded domain M_1 and a compact simply connected homogeneous complex manifold M_2.*

THEOREM B. *Conversely, let M_1 be a homogeneous bounded domain and let M_2 be a compact simply connected homogeneous complex manifold. Let G be a connected Lie group acting on $M_1 \times M_2$ transitively, effectively and holomorphically. Assume further that $M_1 \times M_2$ admits a G-invariant Kähler metric. Then the Lie algebra of G admits a structure of an effective j-algebra so that the associated homogeneous complex manifold coincides with $M_1 \times M_2$.*

Gindikin, Pyatetskii-Shapiro and Vinberg [17] stated that Theorem A was essentially proved in [16]. But it seems to the author that there is no