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The Initial Boundary Value Problem for the Equations of Ideal Magneto-Hydrodynamics

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§ 1. Introduction and Results.

We consider the initial boundary value problem for the equations of ideal magneto-hydrodynamics:

$$(1.1)\quad \begin{cases} (a)\ \rho_p(\partial_t+(u\cdot\nabla))p+\rho \operatorname{div} u=0 \\ (b)\ \rho(\partial_t+(u\cdot\nabla))u+\nabla p+\mu H\times \operatorname{curl} H=0 \\ (c)\ \partial_t H-\operatorname{curl}(u\times H)=0 \end{cases} \quad \text{in } [0, T]\times\Omega,$$

$$(1.2)\quad (p, u, H)|_{t=0}=(p_0, u_0, H_0) \quad \text{in } \Omega,$$

$$(1.3)\quad u\cdot n=0,\ H\times n=g \quad \text{on } [0, T]\times\Gamma.$$

Here Ω is a bounded domain in R^3 with C^∞ boundary Γ, T a given positive constant and $n=n(x)={}^t(n_1, n_2, n_3)$ denotes the unit outward normal at $x\in\Gamma$. Pressure $p=p(t, x)$, velocity $u=u(t, x)={}^t(u_1, u_2, u_3)$ and the magnetic field $H=H(t, x)={}^t(H_1, H_2, H_3)$ are unknowns. The permeability μ is supposed to be constant. We also suppose that density $\rho>0$ is a smooth known function of $p>0$ i.e. $\rho=\rho(p)$; and $\rho_p=\partial\rho/\partial p$. $g=g(t, x)={}^t(g_1, g_2, g_3)$ is a given function on $[0, T]\times\Gamma$. $\partial_t=\partial/\partial t$, $\partial_i=\partial/\partial x_i (i=1,2,3)$, $\nabla=(\partial_1, \partial_2, \partial_3)$, $(u\cdot\nabla)=\sum_{i=1}^{3} u_i\cdot\partial_i$ and $\cdot,\times$ denote scalar and vector product, respectively.

We assume that the initial data p_0 and H_0 satisfy

$$(1.4)\quad \inf_{x\in\Omega}\{\rho(p_0),\ \rho_p(p_0)\}\geqq c_1>0,$$

$$(1.5)\quad \operatorname{div} H_0=0 \text{ in } \Omega,$$

$$(1.6)\quad \inf_{x\in\Gamma}|H_0\cdot n|\geqq c_2>0.$$

Here c_1 and c_2 are positive constants. The assumptions (1.4), (1.5) guarantee the equations (1.1) to be a quasilinear symmetric hyperbolic system.

Our purpose of this paper is to show a local in time existence theorem for the initial boundary value problem (1.1)-(1.3).

THEOREM. *Let m be an integer $\geqq 3$. Assume that $g\in \dot{Y}_m(T)$ and the*