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On H-separable extensions of primitive rings

In memory of Professor Akira Hattori

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Introduction. Throughout this paper every ring will have the identity, and every subring of it will contain the identity of it. A ring is said to be strongly primitive if it has a faithful minimal left ideal. The structure of strongly primitive ring was researched in [1] and [2] by Nakayama and Azumaya. The aim of this paper is to give a necessary and sufficient condition for an H-separable extension ring A of a strongly primitive ring B to be strongly primitive. We will show that, if B is a strongly primitive ring with the socle $\mathfrak{z}$, and if A is an H-separable extension of B such that A is left (or right) B-finitely generated projective, then the necessary and sufficient condition for A to be strongly primitive is that $A\mathfrak{z}A\cap B=\mathfrak{z}$ holds (Theorem 1). This condition is a sufficient condition, if we assume that A is an H-separable extension of a strongly primitive ring B such that B is a left (or right) B-direct summand of A. Finally, we will consider the case where A is a left full linear ring with the center C, D is a simple C-subalgebra of A with $[D:C]<\infty$ and $B=V_A(D)$, the centralizer of D in A. In the above situation Nakayama and Azumaya obtained much more interesting results in [1] and [2]. In particular, they showed that B is also a left full linear ring, $V_A(B)=D$ and that the same inner Galois theory as in simple artinian ring holds in this case, too. In this paper we will show that $S=A\mathfrak{z}A$, $A\mathfrak{z}A\cap B=\mathfrak{z}$ and $S=Soc({}_BA)=Soc(A_B)=A\mathfrak{z}=\mathfrak{z}A$ hold if A and B are in the above situation, where S and $\mathfrak{z}$ are the socles of A and B, respectively (Theorem 2).

Preliminaries. First we recall some definitions. Let A be a ring. Hereafter we will call each two sided ideal of A, simply, an ideal of A. The socle of a left (resp. right) A-module M is the sum of all minimal A-submodules of M, and denoted by $Soc({}_AM)$ (resp. $Soc(M_A)$). A is said to be a left primitive ring if A has a faithful simple left A-module. A right primitive ring is similarly defined, and a both left and right primitive ring is called simply primitive ring. Now we put a stronger condition on A. A is said to be strongly primitive if A has a faithful minimal left ideal. In this case A has also a faithful minimal right ideal. Thus strong primitivity is left