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Some remarks on the Dirichlet problem for semi-linear elliptic equations with the Ambrosetti-Prodi conditions.

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1. Introduction.

In this paper we investigate the solvability of the Dirichlet problem for semi-linear equation

$$(1_s)\qquad Lu=-\sum_{i,j=1}^{n}D_i(a_{ij}(x)D_ju)=f(u)+s\theta(x)+h(x)\ \ in\ Q,$$

$$(2_t)\qquad u(x)=t\phi(x)\ \ on\ \partial Q,$$

in a bounded domain $Q\subset \boldsymbol{R}^n$, with the boundary ∂Q of class C^2, where s and t are real parameters, θ is the first eigenfunction of L and $\theta\perp h$.

In the case, where $t=0$ anf f satisfies the Ambrosetti-Prodi conditions

$$(3)\qquad \lim_{t\to-\infty}\frac{f(t)}{t}<\lambda_1<\lim_{t\to\infty}\frac{f(t)}{t},$$

the problem (1_s), (2_0) has an extensive literature (see [1], [2], [3], [8], [10], [12], [13] and [14]). Here λ_1 denotes the first eigenvalue of L. In these papers, under suitable regularity assumptions on a_{ij} $(i, j=1, \ldots, n)$ f and h, the following result was established. There exists a constant s_0 such that the problem (1_s), (2_0) has 2, 1 or 0 solutions depending on whether s is less than, equal to or greater than s_0.

The purpose of this article is to investigate the dependence of the existence of solutions of (1_s), (2_t) on a parameter t.

The main result can be summarized as follows. Suppose that ϕ is sufficiently smooth, $\phi\geq 0$ and $\phi\not\equiv 0$ on ∂Q. Then there exists a number $s_0=s_0(h, \phi, f)$ such that for every $s\leq s_0$ there exists $t^*(s)$ such that for $t<t^*(s)$ the problem (1_s), (2_t) has at least one solution and no solution for $t>t^*(s)$.

2. Preliminaries.

Throughout this paper we make the following assumptions:

(A) There exists a constant $\gamma>0$ such that