Hokkaido Mathematical Journal Vol. 19 (1990) p. 403~415

' *t*-designs ' in $H(d, q)$

Hiroshi SUZUKI

(Received February 3, 1989)

Abatract

We define two kinds of ' t-designs ' in $H(d, q)$, which is a semilattice of all partial mappings from a d-element set to a q-element set, and prove Fisher type inequalities for those ' t-designs '. They are generalizations of the Ray-Chaudhuri and Wilson inequality for (combinatorial) t-designs and the Rao bound for orthogonal arrays of strength t. We give examples of ' t-designs ' which attain those bounds.

1. Introduction

Interesting similarities between (combinatorial) t-designs and orthogonal arrays have been pointed out by several authors. For example, P. Delsarte defined a concept of regular semilattices and t-designs in them ([2]) and now those two types of designs, namely, (combinatorial) t-designs and orthogonal arrays are understood as examples of t-designs in regular semilattices or those in Q-polynomial association schemes. We consider Hamming type (or hypercubic-type) regular semilattices and define two types of ' t-designs ', namely, $[t]$-designs and $\{t\}$-designs, both of which are generalizations of those two classical-type designs. See Definition 2.2. The concept of $[t]$-designs seems to be first introduced and studied by H. Nagao and others ([1], [4]. See Corollary 3.3).

In this paper we give Fisher type inequalities for two kinds of ' t-designs '. As special cases they include the Ray-Chaudhuri and Wilson inequality for (combinatorial) t-designs and the Rao bound for orthognal arrays. In the final section we give several constructions of ' t-designs ' and also give a series of examples which attain the bound of the Fisher-type inequality. Our method of proof is standard and uses higher incidence matrices, so in that sense it follows the method of R. Wilson [5].

2. ' *t*-designs ' and its incidence matrices

We begin with the definition of a semilattice $H(d, q)$. Throughout this paper ' t-designs ' are considered in this semilattice unless we specify.