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The configurations of the M-curves of degree (4, 4) in $RP^1 \times RP^1$ and periods of real $K3$ surfaces

Dedicated to Professor Haruo Suzuki on his 60th birthday

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Abstract. For M-curves of degree (4, 4) in $RP^1 \times RP^1$ whose components are all contractible, it is known that three configuration types are possible. We prove that all these configuration types are realized by some M-curves of degree (4, 4) by means of the existence of locally universal families of real $K3$ surfaces and the local surjectivity of period mappings defined over those families.

0. Introduction.

We consider the zero set RA of a real homogeneous polynomial F $(\neq 0)$ of degree (d, r) in $RP^1 \times RP^1$, where d and r are integers (≥ 1). We assume that the zero set A of F in $CP^1 \times CP^1$ is nonsingular. (In what follows, we write $P^1 \times P^1$ for $CP^1 \times CP^1$.) Then A is a connected complex 1-dimensional manifold. But RA is a possibly disconnected real 1-dimensional manifold (a disjoint union of finitely many copies of S^1) or the empty set. It is known that the number of the connected components of RA does not exceed $(d-1)(r-1)+1$(see [**5**]). We remark that the number $(d-1)(r-1)$ is the genus of the nonsingular curve A. We say RA is an *M-curve* of degree (d, r) if it has precisely $(d-1)(r-1)+1$ connected components.

In this paper we make clear the "configurations" of the M-curves of degree (4, 4) in $RP^1 \times RP^1$, where we consider only the curves whose components (embedded S^1) are all contractible in $RP^1 \times RP^1$. We define the meaning of the "configurations" as follows. In our cases, each component of RA, which is called an *oval*, divides $RP^1 \times RP^1$ into two connected components. One of those is homeomorphic to an open disk and called the *interior* of the oval. The other is called the *exterior* of that. As a consequence of [**5**], every M-curve of degree (4, 4) lies in one of the following three cases (cf. Figure 1).

(1) Each of certain 9 ovals lies in the exteriors of the others, and the interior of one of those contains one oval. $\left(\text{Notation}: \frac{1}{1} 8\right)$