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The F. and M. Riesz theorem on certain transformation groups, II

by

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§1. Introduction.

The classical F. and M. Riesz theorem was extended, by Helson-Lowdenslager and deLeeuw-Glicksberg, to compact abelian groups with ordered duals. As an extension of the result of deLeeuw and Glicksberg, Forelli extended the F. and M. Riesz theorem to a (topological) transformation group in which the reals $\boldsymbol{R}$ acts on a locally compact Hausdorff space.

On the other hand, the author ([14]) obtained several results, corresponding to Forelli's theorems, on a (topological) transformation group in which a compact abelian group acts on a locally compact Hausdorff space under certain conditions. In fact, the author obtained the following in [14].

THEOREM 1.1 (cf. [14, Theorem 1.1]). *Let (G, X) be a transformation group in which G is a compact abelian and X is a locally compact Hausdorff space. Suppose (G, X) satisfies conditions (C. I) and (C. II) (see* [14]). *Let P be a semigroup in $\hat{G}$ such that $P\cup(-P)=\hat{G}$. Let σ be a positive Radon measure on X that is quasi-invariant. Let $\mu\in M(X)$, and let $\mu=\mu_a+\mu_s$ be the Lebesgue decomposition of μ with respect to σ. Suppose* $\mathrm{sp}(\mu)\subset P$. *Then both* $\mathrm{sp}(\mu_a)$ *and* $\mathrm{sp}(\mu_s)$ *are contained in P. If, in addition, $P\cap(-P)=\{0\}$ and $\pi(|\mu|)\ll\pi(\sigma)$, then* $\mathrm{sp}(\mu_s)\subset P\backslash\{0\}$, *where $\pi: X\rightarrow X/G$ is the canonical map.*

THEOREM 1.2 (cf. [14, Theorem 1.2]). *Let (G, X) be as in Theorem 1.1. Let E be a subset of $\hat{G}$ satifying the following :*

(*) *For any nonzero measure $\lambda\in M_E(G)$, $|\lambda|$ and m_G are mutually absolutely continuous.*

Let μ be a measure in $M(X)$ with $\mathrm{sp}(\mu)\subset E$. *Then μ is quasi-invariant.*

THEOREM 1.3 (cf. [14, Theorem 1.3]). *Let (G, X) be as in Theorem 1.1. Let E be a Riesz set in $\hat{G}$. Let μ be a measure in $M(X)$ with*