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Boundedness of minimizers

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Abstract. We find conditions guaranteeing that solutions to typical problems of the calculus of variations are bounded.
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§ 0 Introduction

Consider a variational problem of the following type:

$$(0.0)\qquad \begin{cases} \int_G f(u, Du)\,dx = \text{minimum}, \\ \text{under the condition: } u = g \text{ on } \partial G. \end{cases}$$

Here G, f, g are given; G is an open subset of euclidean space $\boldsymbol{R}^n$; n, the dimension, is greater than 1; competing functions u are assumed to have scalar values. D stands for gradient and $dx = dx_1 \ldots dx_n$, the Lebesgue n-dimensional measure.

We address ourselves to the following question. Suppose a minimizer exists and g, the boundary datum is bounded. Is such a minimizer bounded?

An approach to this question is using the Euler equation of problem (0.0) and developing maximum principles for weak solutions to nonlinear partial differential equations of elliptic type — one aim of an earlier paper [Tal]. This approach requires some smoothness of integrand f and essentially involves the first order derivatives of f.

In the present paper we merely assume a condition on the growth of integrand $f(u, \xi)$ with respect to the last variable ξ. In its simplest form, such a condition reads:

$$f(u, \xi) \geq A(|\xi|)$$

for every scalar u and any vector ξ in $\boldsymbol{R}^n$. More generally, we assume

$$(0.1)\qquad f(u, \xi) \geq A(|\xi|) - A(\lambda |u|).$$

Here A is some Young function (see section 1(i)) and λ is some nonnegative constant; $|\xi| = (\xi_1^2 + \ldots + \xi_n^2)^{1/2}$, the length of ξ.

Let B be any increasing function such that