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Relative bounds of closable operators in non-reflexive Banach spaces

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Introduction

In this paper we discuss some perturbation problems related to the relative compactness and boundedness of closable operators in complex Banach spaces which are *not necessarily reflexive.*

Let X, Y and Z be Banach spaces, let A be an operator from X into Z and let B be an operator from X into Y with $D(A)\subset D(B)$, where $D(T)$ denotes the domain of an operator T. We consider the following three conditions (see T. Kato [3] and S. G. Krein [4]):

(I) B is A-compact, i.e., for any sequence $\{u_{n}\}$ in $D(A)$ with $\sup_{n\in N}(\|u_{n}\|_{X}+\|Au_{n}\|_{Z})<\infty$, $\{Bu_{n}\}$ has a convergent subsequence $\{Bu_{n_{j}}\}$ in Y.

(II) B is subordinate to A with exponent $\alpha\in(0,1)$, i.e., there is a constant C_{α} such that for all $u\in D(A)$

$$\|Bu\|_{Y}\leq C_{\alpha}\|Au\|_{Z}^{\alpha}\|u\|_{X}^{1-\alpha}.$$

(III) B is A-bounded with A-bound zero, i.e., for any $\varepsilon>0$ there is a constant C_{ε} such that for all $u\in D(A)$

$$\|Bu\|_{Y}\leq\varepsilon\|Au\|_{Z}+C_{\varepsilon}\|u\|_{X}.$$

It is clear that (II) implies (III). P. Hess [1][2] has proved that (I) implies (III) in the case $X=Y=Z$, where X is *reflexive* and A is *closed*. He has also observed that both reflexivity of X and closedness of A are necessary. M. Schechter [6] has proved that (I) implies (III) in the case $X=Y=Z$, where X is *not necessarily reflexive*, A is *closed*, and B is *closable*.

In § 1 we prove that exen when X, Y, Z are not reflexive and A is not closed, (I) implies (III) under the condition that B is closable, which is also shown not removable. Moreover, we prove that there exist a Banach space X, a closed operator A and a non-closable operator B in X satisfying (I) and (II). Furthermore, we prove that there exist a Banach space X and closed operators A, B in X such that (II) does not hold for any $\alpha\in(0,1)$ but (I) holds. Let $X=Y=Z=L^{2}(\boldsymbol{R}^{n})$ and let