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Holonomy groupoids of generalized foliations, I

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Introduction

We mean by a generalized foliation, a foliation with singular leaves in the sense of P. Stefan [St] and P. Dazord [D], and in the present paper we establish a notion of a holonomy groupoid of this generalized foliation. In order to keep similarities to the case of regular foliations, we set a limitation on singular leaves, however our result is applicable to some foliations of Poisson structures and to some foliations which are not locally simple (cf. [E]). In many cases, we call a generalized foliation simply a foliation.

According to [D], along a leaf F of a foliation $\mathscr{D}$ on a C^{∞}-manifold M there is a unique germ Δ_F of transverse structure. A singular leaf F is called *tractable* if F has a saturated neighborhood N in M with following properties:

(i) N is isomorphic to a fibre bundle over F, $\pi_F: N\rightarrow F$ having a fibre V with a foliation Δ_V which is a representative of Δ_F.

(ii) The structural group of the bundle is the group of isomorphisms of Δ_V and the foliation of N determined by a local product of the one leaf foliation of F and Δ_V is the restriction $\mathscr{D}_N$ of $\mathscr{D}$ to N.

We will show that if all singular leaf of $\mathscr{D}$ is tractable, then an (algebraic) holonomy groupoid $G(\mathscr{D})$ of D is defined and we have

THEOREM 2.2 *If each singular leaf of $\mathscr{D}$ is tractable, then $G(\mathscr{D})$ is a topological groupoid.*

The part of $G(\mathscr{D})$ outside singular leaves is a (non-Hausdorff) C^{∞}-manifold by the usual theory of regular foliations (see, e.g., [P], [Wi]), but $G(\mathscr{D})$ itself is not a manifold in general. We take examples of the foliations from those of symplectic leaves of Poisson structures and from some constructions of fibre bundles. In his construction of a singular foliation C^{*}-algebra, A. Sheu [Sh] uses the notion of holonomy of a locally simple foliation due to C. Ehresmann [E], but there are some foliations which are not locally simple. Our definition of holonomy can be applied to these.

In Section 1, we explain properties of a (generalized) foliation $\mathscr{D}$. In