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On H-separable extensions of primitive rings II

Dedicated to Professor Kazuhiko Hirata on his 60th birthday

Kozo SUGANO

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Introduction. Throughout this paper every ring is assumed to have the identity, and all subrings of a ring will contain the identity of the ring, unless otherwise stated. Let B be a strongly primitive ring and A an H-separable extension of B, and suppose A is left B-finitely generated projective. In [13] it is shown that in this case A is also strongly primitive if and only if $A\mathfrak{z}A\cap B=\mathfrak{z}$, where $\mathfrak{z}$ is the socle of B. The aim of this paper is to detail the structure of A and B which satisfy the above condition. Let furthermore I and $\mathfrak{m}$ be faithful minimal left ideals of A and B, respectively, and denote the double centralizers of ${}_AI$, ${}_BI$ and ${}_B\mathfrak{m}$ by A^*, $\tilde{B}$ and B^*, respectively. Then there exists a ring isomorphism Φ of B^* to $\tilde{B}(\subseteq A^*)$ such that $\Phi(b)=b$ for each $b\in B$, and A^* is an H-separable extension of $\tilde{B}(\cong B^*)$ (Theorem 3.3), that is, the right full linear ring A^* is an inner Galois extension of the right full linear ring B^* (See Theorem 4 [11]). We will also treat the inner Galois theory of full linear rings in §4. Let A be a right full linear ring with its center C, D a simple C-subalgebra of A with $[D:C]<\infty$ and $B=V_A(D)$. Denote the class of right full linear subrings R of A such that R contains B and the simple left ideal of A is a finite direct sum of faithful simple left R-modules by $\mathscr{L}$, and the class of simple C-subalgebras of $V_A(B)$ by $\mathscr{D}$. We already know that there exists a duality between $\mathscr{L}$ and $\mathscr{D}$. We will show that a right full linear subring R of A containing B is in $\mathscr{L}$ if and only if A is left or right B-projective (Theorem 4.1). §1 is the preparation for §2, and in §2 we will introduce some fundamental properties of strongly primitive rings. Let R be a ring and M a flat left R-module, and denote the Gabriel topology of R consisting of right ideals $\mathfrak{a}$ of R such that $\mathfrak{a}M=M$ by $\mathfrak{F}$. As K. Morita showed in [5], there is a ring isomorphism θ of $R_{\mathfrak{F}}$, the ring of quotients of R with respect to $\mathfrak{F}$, to a subring of $R^*=\mathrm{Bic}({}_RM)$ In [3] the author gave a simpler proof of this theorem. Here we will determine $\mathrm{Im}\,\theta$ completely, and show that $\mathrm{Im}\,\theta$ consists of elements r^* of R^* such that $\mathfrak{a}r^*\subset\tilde{R}$ for some $\mathfrak{a}$ in $\mathfrak{F}$, where $\tilde{R}$ is the image of the canonical map of R to R^* (Theorem 1.1). By applying this theorem to