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Second order hyperbolic equations with time-dependent singularity or degeneracy

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Introduction

Let H be a Hilbert space with norm $\|\cdot\|$, and let Λ be a non-negative self-adjoint operator in H. Let S_1, S_2, t_0, α and ν be real numbers with $S_1 \leqq 0 \leqq S_2$, $S_1 \leqq t_0 \leqq S_2$, $\alpha' > -1$ and $-2\alpha - 1 < \nu < 1$. We are concerned with the well-posedness of the following singular or degenerate hyperbolic equation in H:

$$\left.\begin{array}{ll}(0.1) & u''(t) + \phi^2(t)\Lambda u(t) + \psi(t)u'(t) + \Xi(t)u(t) = f(t) \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{on } (t_0, S_2), \\ (0.2) & u(t_0) = u_0, \ |t|^\nu u'(t)|_{t=t_0} = u_1, \end{array}\right\} \text{(WE)}$$

where u' is the t-derivative in the sense of vector-valued derivative, ϕ and ψ are functions on $[S_1, S_2]$ to $[0, +\infty]$ satisfying the following;

$$(0.3) \qquad \phi(\cdot) \in W^{2,\infty}_{\mathrm{loc}}((S_1, S_2)\backslash\{0\}),$$

$$(0.4) \qquad C^{-1}|t|^\alpha \leqq \phi(t) \leqq C|t|^\alpha,$$

$$(0.5) \qquad |\phi'(t)| \leqq C|t|^{\alpha-1}, \ |\phi'(t)| \leqq C|t|^{\alpha-2},$$

for a. e. t on (S_1, S_2), with some positive constant C,

$$(0.6) \qquad \psi(t) - \nu/t \in L^1(S_1, S_2),$$

We note that ϕ takes value 0 or ∞ at $t=0$. That is, the singularity or the degeneracy of (0.1) occurs at $t=0$, which may be initial time ($t_0=0$) or not ($t_0 \neq 0$). Especially if $2\alpha > -1$, we can take $\nu = 0$. In [15], we showed the well-posednes of (WE) in the space $H = L^2(\Omega)$, where Ω is a bounded domain in $\boldsymbol{R}^n$ with smooth boundary, $\Lambda = -\Delta$ with homogeneous Dirichlet boundary condition, $2\alpha > -1$, $\nu = 0$, $\phi(t) = t^\alpha$, $\psi = f = 0$, $\Xi = 0$. The purpose of this paper is to generalize the above theorem. For this purpose, we first prove an abstract theorem on the well-posedness of non-homogeneous evolution equation, which generalizes the abstract theorem on that of homogeneous equation in [15] (see Theorem 2). Then we solve (WE) by applying this abstract theorem (see Theorem 1).

Equation (0.1) with $t_0=0$ is studied by various authors: see Carroll-