Hokkaido Mathematical Journal Vol. 20 (1991) p. 579-591

Algebras A_p and B_p and amenability of locally compact groups

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(Received July 16, 1990)

0. Introduction.

Since the appearance of the pioneer work of Eymard [4], the Fourier algebra $A(G)$ of a locally compact group G has been studied by many authors in connection with the theory of unitary representations and the theory of operator algebras. As related algebras, the algebras $A_p(G)$ and the algebras $B_p(G)$ of Herz-Schur multipliers for $1<p<\infty$ have been investigated by Eymard [5] and Herz [8-10] together with the algebras $PF_p(G)$ of pseudofunctions and $PM_p(G)$ of pseudomeasures. Remark here that $A(G)=A_2(G)$. In general the algebra $A_p(G)$ is contractively imbedded in $B_p(G)$. When G is amenable, this imbedding is isometric.

It is shown in [8, 11] that if the group G is amenable, then $A_{p'}(G)$ is contractively included in $A_p(G)$ whenever $1<p<p'\leq 2$ or $2\leq p'<p<\infty$. In particular, $A(G)$ is contractively included in every $A_p(G)$. It is known that the same relation holds also for $B_p(G)$ (see Remark 2.5 (1)). However, according to Pytlik [18], we know that when $\boldsymbol{F}_r$ is a free group with r generators, $2\leq r\leq\infty$, a typical example of non-amenable groups, for any distinct pair p, p' there does not exist any inclusion relation between $A_p(\boldsymbol{F}_r)$ and $A_{p'}(\boldsymbol{F}_r)$ (see Remark 2.5 (2)). In section 2, we will prove that for every locally compact group G the algebra $B_2(G)$ is contractively included in $B_p(G)$. As a consequence, we show that when $\boldsymbol{F}_r$ is a free group, for any $1<p<\infty$ the algebra $A_p(\boldsymbol{F}_r)$ has an approximate identity $\{u_n\}$ such that $\sup_n\|u_n\|_{B_p}\leq 1$. This result should be compared with the well-known result (e.g. [9], [15]) that $A_p(G)$ has a bounded approximate identity if and only if the group G is amenable.

Nebbia [16] characterized the amenability of G in terms of multipliers of $A(G)$ into the space $M(G)$ of finite complex Radon measures or $L^1(G)$. In section 3, for $1<p<\infty$ and $1\leq p'<\infty$, we define multipliers of $A_p(G)$ into $M(G)$ or $L^{p'}(G)$, and those of $W_p(G)$ (the dual space of $PF_p(G)$) into $M(G)$ or $L^{p'}(G)$. For instance, a multiplier of $A_p(G)$ into $M(G)$ is a bounded linear operator $\Phi: A_p(G)\rightarrow M(G)$ such that $\Phi(uv)=u\Phi(v)$ for all $u, v\in A_p(G)$. Any element of $M(G)$ defines a multiplier in natural