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On groups G of p-length 2 whose nilpotency indices of J(KG) are a(p-1)+1

Dedicated to Professor Tosiro TSUZUKU on his 60th birthday

Hiroshi FUKUSHIMA
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1. Introduction

Let G be a finite p-solvable group with a Sylow p-subgroup P of order p^{a}, K a field of characteristic p, KG the group algebra of G over K, and $t(G)$ the nilpotency index of the radical $J(KG)$ of KG.

D. A. R. Wallace [9] proved that $a(p-1)+1\leqq t(G)\leqq p^{a}$. Y. Tsushima [8] proved that the second equality $t(G)=p^{a}$ holds if and only if P is cyclic. Here we shall study the structure of G with $t(G)=a(p-1)+1$. If G has p-length 1, then $t(G)=t(P)$ by Clarke [1]. From this, we can easily see that $t(G)=a(p-1)+1$ if and only if P is elementary abelian. Therefore we shall be interested in the structure of G of p-length 2 with $t(G)=a(p\text{-}1)+1$. As such examples, we know the followings.

We set $q=p^{r}$ and $l=(q^{p}-1)/(q-1)$. Then $q-1$ and l are relatively prime. Let $F=GF(q^{p})$ be a finite field of q^{p} elements, λ a generator of the multiplicative group F^{*} of F, and $\nu=\lambda^{q-1}$. Let V be the additive group of F. If we define $v^{x}=\nu v$, where νv means a multiplication in the field F, then $x\in \mathrm{Aut}(V)$. Let U be the Galois group of F over $GF(q)$, and $H=\langle x\rangle$. Then $HU\subseteqq \mathrm{Aut}(V)$. So we can consider the semidirect product of V by HU. We set $M_{p,r}=VHU$. Then HU is a Frobenius group and $|H|=l$, $|U|=p$, and VU is a Sylow p-subgroup of $M_{p,r}$ of order p^{pr+1}. In [5], Motose proved $t(M_{p,r})=(pr+1)(p-1)+1$.

Let $G=M_{p,r}$, then $G=O_{p,p',p}(G)$ and $G/O_{p}(G)$ is a Frobenius group. So can we consider conversely that if G satisfies such conditions and $t(G)=a(p-1)+1$, then is G isomorphic to $M_{p,r}$? Concerning this problem, we have the following result.

THEOREM. *Let V be a normal p-subgroup of G with $G=VN$ and $V\cap N=1$ for some Frobenius group N with complement U and kernel H, where U and H are p-group and abelian p'-group, respectively. Then the*