Hokkaido Mathematical Journal Vol. 20 (1991) p. 519-521

A note on the Lonergan-Hosack presentation

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(Received March 27, 1990)

Introduction.

The problem of finding different methods to prove groups finite or infinite was discussed by M. Newman in a lecture which he gave at the GROUPS-KOREA 1988 conference in Pusan. As an example he considered the Lonergan-Hosack presentation

$$G(m)=\langle x, z: z^{3}x^{m-4}z^{3}x^{-1}=z^{5}x^{m-3}z^{2}x^{m-3}=1\rangle.$$

For the two cases $m=1$ and $m=5$ we have fundamental group presentations of closed, orientable 3-dimensional manifolds. For a while it was an open problem whether these presentations are those of finite or infinite groups. The first solution came from M. Slattery [**4**] who managed to show that $G(1)$ and $G(5)$ are infinite by using the computer algebra package CAYLEY. In his conference talk M. Newman welcomed all contributions to this area and the purpose of this note is to show that the use of some supporting theory provides us with more information about the structure of $G(m)$.

Results.

Consider the presentation $G(m)$ given in the introduction.

THEOREM 1. *Let $m\geq 5$ or $m=3$ or $m<0$. Then*

(1) *$G(m)$ has a subgroup of finite index mapping onto a free group of rank 2 and $G(m)$ has a free subgroup of rank 2.*

(2) *$G(m)$ has a generating pair $\{u, v\}$ such that the subgroup generated by elements u^{k} and v^{k} is free of rank 2 for a sufficiently large integer k.*

(3) *$G(m)$ is SQ-universal.*

PROOF: If $m=5$, then the result follows directly from [**3**]. If $m=3$, then $G(3)\cong Z_{2}\star Z_{7}$ and the result is well known (see [**2**]). Then assume

The authors are grateful to the Korea Science and Engineering Foundation, without whose Conference support 1988 this collaboration would not have been possible.