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Contact surgery and symplectic handlebodies

For Professor Noboru Tanaka
on his 60th birthday

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1. Introduction

The construction and classification of contact manifolds is a basic problem in differential topology. It was shown by Meckert in [5] that the connected sum of two contact manifolds carries a contact structure. (See [8] for applications of Meckert's theorem.) Since the connected sum of two contact manifolds is obtained from their disjoint union (which obviously carries a contact structure) by a simple form of elementary surgery, it is natural to try to extend Meckert's results to more general surgeries. The present paper provides such an extension, while simplifying Meckert's construction as well.

Let X be an orientable contact manifold with contact distribution $\mathscr{D}\subset TX$. $\mathscr{D}$ may be defined by a 1-form α for which $\mathscr{D}=\ker\alpha$ and $d\alpha$ is nondegenerate on $\mathscr{D}$. Such an α is called a *contact form* for the contact structure. The symplectic structure on $\mathscr{D}$ defined by $d\alpha$ is multiplied by a function when α is, and so the vector bundle $\mathscr{D}$ has a natural conformal symplectic structure; in particular, there is a well defined "symplectic orthogonal" operation $\perp'$ on subbundles of $\mathscr{D}$.

A submanifold Y of X is called *isotropic* if all its tangent spaces are contained in $\mathscr{D}$. Since any contact form α vanishes on Y, so does $d\alpha$, so that TY is contained in $(TY)^{\perp'}$. The quotient $(TY)^{\perp'}/TY$ carries a conformal symplectic structure and is called the (conformal) *symplectic normal bundle* of Y. We denote it by $CSN(X, Y)$. The ordinary normal bundle $N(X, Y)=T_YX/TY$ of Y in X is isomorphic to the direct sum of $CSN(X, Y)$, the trivial line bundle $T_YX/\mathscr{D}_Y$, and the quotient $\mathscr{D}_Y/(TY)^{\perp'}$. The last bundle is naturally isomorphic to T^*Y, so if we have a trivial-

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