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A property of spectrums of measures on certain transformation groups

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§1 Introduction.

Let X be a locally compact Hausdorff space. Let $C_0(X)$ be the Banach space of continuous functions on X which vanish at infinity, and let $M(X)$ be the Banach space of complex-valued bounded regular Borel measures on X with the total variation norm. Let $M^+(X)$ be the set of nonnegative measures in $M(X)$. For $\mu\in M(X)$ and $f\in L^1(|\mu|)$, we often write $\mu(f)=\int_X f(x)d\mu(x)$. Let X' be another locally compact Hausdorff space, and let $S: X\to X'$ be a continuous map. For $\mu\in M(X)$, let $S(\mu)\in M(X')$ be the continuous image of μ under S. We denote by $\mathscr{B}(X)$ the σ-algebra of Borel sets in X. $\mathscr{B}_0(X)$ means the σ-algebra of Baire sets in X. That is, $\mathscr{B}_0(X)$ is the σ-algebra generated by compact G_δ-sets in X.

Let G be a LCA group with dual $\hat{G}$. $M(G)$ and $L^1(G)$ denote the measure algebra and the group algebra respectively. For $\mu\in M(G)$, $\hat{\mu}$ denotes the Fourier-Stieltjes transform of μ. m_G denotes the Haar measure of G. Let $M_a(G)$ be the set of measures in $M(G)$ which are absolutely continuous with respect to m_G. Then by the Radon-Nikodym theorem we can identify $M_a(G)$ with $L^1(G)$. For a subset E of $\hat{G}$, $M_E(G)$ denotes the space of measures in $M(G)$ whose Fourier-Stieltjes transforms vanish off E. For a closed subgroup H of G, $H^\perp$ stands for the annihilator of H.

Let (G, X) be a (topological) transformation group, in which G is a compact abelian group and X is a locally compact Hausdorff space. That is, suppose that there exists a continuous map $(g, x)\to g\cdot x$ from $G\times X$ onto X with the following properties:

(1.1) $x\to g\cdot x$ is a homeomorphism on X for each $g\in G$ and $0\cdot x=x$, where 0 is the identity element in G;

(1.2) $g_1\cdot(g_2\cdot x)=(g_1+g_2)\cdot x$ for g_1, $g_2\in G$ and $x\in X$.

We note that $(g, x)\to f(g\cdot x)$ is a Baire function on $G\times X$ for each Baire function f on X. For $\lambda\in M(G)$ and $\mu\in M(X)$, define $\lambda*\mu\in M(X)$ by