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p-supersolvability of factorized finite groups

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1. Introduction

All groups we consider are finite. It is well known that the product of two supersolvable normal subgroups is not supersolvable in general (see Huppert [3]).

In [2]. Baer proved that if G is the product of two supersolvable normal subgroups and the commutator subgroup G' of G is nilpotent, then G is supersolvable.

In [1]. Asaad and Shaalan proved the following generalization of Baer's theorem:

Suppose that H and K are supersolvable subgroups of G, G' is nilpotent and $G=HK$. Suppose further that H is permutable with every subgroup of K and K is permutable with every subgroup of H. Then G is supersolvable.

Further, they proved the following result:

Suppose that H is a nilpotent, K a supersolvable subgroup of G and $G=HK$. Suppose further that H is permutable with every subgroup of K and K is permutable with every subgroup of H. Then G is supersolvable.

If H and K are subgroups of a group G such that H is permutable with every subgroup of K and K is permutable with every subgroup of H, we say that H and K are mutually permutable and we say that H and K are totally permutable if every subgroup of H is permutable with every subgroup of K.

The purpose of the present communication is the presentation of some properties of products of mutually permutable subgroups:

THEOREM A. *Let $G=HK>1$ be a group where H and K are mutually permutable. Then H or K contains a nonidentity normal subgroup of G or $\boldsymbol{F}(G)\neq 1$, where $\boldsymbol{F}(G)$ denotes the Fitting subgroup of G.*

Further we present a generalization and give independent proofs of the above mentioned results of Asaad and Shaalan in the following sense: