Hokkaido Mathematical Journal Vol. 21(1992) p. 365-378

Solvability of convolution equations in spaces of generalized distributions with restricted growth

J. IBBOTSON and Z. ZIELEZNY
(Received May 24, 1991)

L. Ehrenpreis [6] has established necessary and sufficient conditions on the Fourier transform of a distribution $S\in\mathscr{E}'(\boldsymbol{R}^{n})$ in order that the convolution equation

(0.1) $\quad S*u=v$

has a solution $u\in\mathscr{D}'(\boldsymbol{R}^{n})$, for every $v\in\mathscr{D}'(\boldsymbol{R}^{n})$, or more briefly, in order that

(0.2) $\quad S*\mathscr{D}'(\boldsymbol{R}^{n})\supset\mathscr{D}'(\boldsymbol{R}^{n})$.

He proved that, for a distribution $S\in\mathscr{E}'(\boldsymbol{R}^{n})$, (0.2) is valid if and only if there are positive constants A_{1}, A_{2}, and A_{3} such that for every $\xi\in\boldsymbol{R}^{n}$ there exists an $\eta\in\boldsymbol{R}^{n}$ satisfying the conditions

(0.3) $\quad |\xi-\eta|\leq A_{1}\log(2+|\xi|)$ and $|\hat{S}(\eta)|\geq(A_{2}+|\xi|)^{-A_{3}}$.

In this case S is called invertible.

Later ([1], [2], [4], [5], [8], [11], [13], [14]), other versions of the invertibility conditions (0.3) were used in order to prove the existence of solutions of convolution equations in various spaces of distributions and generalized distributions.

In this paper we study convolution equations in the spaces of generalized distributions of G. Björck [3] with restricted growth. Specifically, we construct a space $\mathscr{K}_{M,\omega}'$ of generalized distributions "growing" no faster than $e^{M(ax)}$, for some $a>0$, where M is a function defined similarly to those used in the definition of the spaces W_{M} in [7]. We then characterize the convolution operators S in $\mathscr{K}_{M,\omega}'$ for which $S*\mathscr{K}_{M,\omega}'\supset\mathscr{K}_{M,\omega}'$; these operators are called (M,ω)-invertible.

In the particular case when $\omega(\xi)=\log(1+|\xi|)$, $\mathscr{K}_{M,\omega}'$ is a space of distribution and our result coincides with that of S. Abdullah [2].

§1. Preliminaries

We use the notation and the basic properties of generalized functions given in [3].