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Interpolating Blaschke products and the left spectrum of multiplication operators on the Bergman space

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Abstract. This paper studies the problem of approximating a Blaschke product by interpolating Blaschke products. We will solve the problem for some special classes of Blaschke products. Then we will give a connection between this problem and the solidity near the origin in the left spectrum of a multiplication operator on the Bergman space.

1. Definitions and notations

Let D be the open unit disk in the complex plane and ∂D be the unit circle. The Banach algebra $H^\infty(D)$ is the algebra of all bounded holomorphic functions on D under the sup-norm topology. The Bergman space $L_a^2(D)$ is the Hilbert space of all holomorphic functions f on D such that

$$\iint_D |f(z)|^2 dA(z) < \infty,$$

where A denotes the area measure of the plane. For $f \in H^\infty(D)$, the multiplication operator M_f on $L_a^2(D)$ is the bounded operator that sends $g \in L_a^2(D)$ to $fg \in L_a^2(D)$.

A *Blaschke product* is a function in $H^\infty(D)$ of the form

$$B(z) = e^{i\theta} \prod_{n=1}^{\infty} \frac{|z_n|}{z_n} \frac{z_n - z}{1 - \overline{z_n} z},$$

where the sequence $\{z_n\}$ is in D and satisfies $\sum_{n=1}^{\infty}(1-|z_n|) < \infty$. (If some $z_n = 0$, then the corresponding factor is to be interpreted as z). A sequence $\{z_n\}$ in D is called an *interpolating sequence* if for each bounded sequence $\{w_n\}$ of complex numbers, there is some $f \in H^\infty(D)$ such that $f(z_n) = w_n$. A Blaschke product is called an *interpolating Blaschke product* (or *ibp* for short) if the sequence $\{z_n\}$ is an interpolating sequence. A well-known open question [6, p. 430] in function theory is whether every Blaschke product can be uniformly approximated by interpolating Blaschke products or not.