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Some properties of Fourier transform for operators on homogeneous Banach spaces

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Abstract.

The Fourier transform of linear operator on a general homogeneous Banach space B in $L^1(G)$ for locally compact abelian group G is defined and characterized. It is proved that the Fourier transform of a linear operator is an operator valued continuous function on $\widehat{G}$, th dual group of G, and vanishing at infinity. Convolution of function and operator is studied. Some linear operator on B is characterized as an integration of its Fourier transform over $\widehat{G}$.

1. Introduction and preliminaries

Throughout the paper let G be a locally compact as well as a σ-compact abelian group, and let $\widehat{G}$ be its dual group. A *homogeneous Banach space* B is a dense subspace of $L^1(G)$ such that

(i) B is a Banach space under another norm $\| \ \|_B$ which is stronger than $L^1(G)$-norm $\| \ \|_1$.
(ii) The norm $\| \ \|_B$ is translation invariant and $\|R_x f-f\|_B \to 0$ as $x \to 0$ in G where $R_x f(y)=f(y-x)$ for all x and y in G.

Some special homogeneous Banach spaces are investigated in Larsen [6], Lai [7-10]. For example, the spaces

$$A^p(G)=\{f \in L^1(G) : \hat{f} \in L^p(\widehat{G}),\ 1 \leq p \leq \infty\}$$

with norm $\|f\|_{A^p(G)}=\|f\|_1+\|\hat{f}\|_p$
and $\quad A_{1,p}(G)=L^1 \cap L^p(G)$ with norm $\|f\|=\|f\|_1+\|f\|_p$

are homogeneous Banach spaces.

A homogeneous Banach space B may not admit multiplication by character $\gamma \in \widehat{G}$, and even if it does, it may not be isometry under the norm $\| \ \|_B$ (see Reiter [15]). If for any $\gamma \in \widehat{G}$, the operator

$$M_\gamma : f \in B \to \gamma \cdot f \in B$$