Hokkaido Mathematical Journal Vol. 21 (1992) p. 251-258

On geometric spectral radius of commuting n-tuples of operators

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(Received October 25, 1990, Revised April 19, 1991)

Dedicated to Professor George Maltese on his 60th birthday

Abstract. The aim of this paper is to prove that for most classical joint spectra as e.g. the Taylor spectrum, the Harte spectrum, Słodkowski spectra, also left and right spectrum, the joint approximate point spectrum, and some other spectroids, the geometric spectral radius is the same and depends only upon a commuting n-tuple of operators. We generalize also this result by showing that for all these spectra or sub-spectra their convex envelopes coincide.

1. Definitions and notation

Let X be a complex Banach space. Denote by $B(X)$ the algebra of all continuous linear operators on X. Put $B_{\mathrm{com}}^{n}(X)$ for the set of all n-tuples of commuting operators in $B(X)$ and put $B_{\mathrm{com}}(X)=\bigcup_{n=1}^{\infty}B_{\mathrm{com}}^{n}(X)$, in particular $B(X)$ identified with $B_{\mathrm{com}}^{1}(X)$ is a subset of $B_{\mathrm{com}}(X)$. Suppose that to each n-tuple $(T_1, \ldots, T_n)$ in $B_{\mathrm{com}}(X)$ there corresponds a subset $\sigma_s(T_1, \ldots, T_n)\subset \boldsymbol{C}^n$, and consider the following axioms (it is a small modification of axioms given by the second author in [11]).

(i) $\sigma_s(T_1, \ldots, T_n)$ is a non-void compact subset of $\boldsymbol{C}^n$, for all $(T_1, \ldots, T_n)\in B_{\mathrm{com}}(X)$,

(ii) $\sigma_s(T_1, \ldots, T_n)\subset\prod_{i=1}^{n}\sigma(T_i)$, where $\sigma(T)$ denotes the usual spectrum of an operator T in $B(X)$, $(T_1, \ldots, T_n)\in B_{\mathrm{com}}(X)$. In particular, for a single operator T we have

$$\sigma_s(T)\subset\sigma(T).$$

The next axiom is the equality in the above formula

This research was partially supported by Grant-in-Aid for Scientific Research, Japanese Mimistry of Education