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An elementary and unified approach to the Mathieu-Witt systems II: The uniqueness of W_{22}, W_{23}, W_{24}

Dedicated to Professor Tosiro Tsuzuku
on his sixtieth birthday

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Abstract.

In this paper we present a new proof of the uniqueness of the large Witt systems W_{22}, W_{23}, W_{24}. Their uniqueness is (almost) simultaneously proved by the same and simple method, and their existence is also shown.

1. Introduction

Although self-contained, this paper is a continuation of our previous article [3], which was intended as the title shows, but made no mention of the uniqueness of the Witt systems. Nowadays not a few proofs of their uniqueness are known (see, e. g. [1], [2], [4], [5, Chap. 20]). The purpose of this paper is to present an alternative, simple, elementary and unfied proof of the uniquenesss of the large Witt systems W_{22}, W_{23}, W_{24}. 'Simple, elementary' means that our proof uses only block intersection property BIP (mentioned later) which is easily shown, not using any knowledge of finite geometry such as projective planes, coding theory, etc. 'Simple, unified' means that the uniqueness of the three systems can be (almost) simultaneously proved by the same method. We note also that our uniqueness proof shows the existence of the three systems (see Remark 3).

DEFINITIONS AND NOTATION. Let Ω be a set of v points and $\mathfrak{B}$ a collection of k-subsets (called blocks) of Ω. The pair $\boldsymbol{D}=(\Omega, \mathfrak{B})$ is called a *t-design* with *parameters* $t, v, k, \lambda (v>k>t>0$ and $\lambda>0)$ or, briefly a *t-(v, k, λ) design* if any t-subset of Ω is contained in exactly λ blocks of $\mathfrak{B}$. If $\boldsymbol{D}=(\Omega, \mathfrak{B})$ is a t-(v, k, λ) design, then, for any $s \leq t$, the number of blocks containing any s-subset of Ω is equal to $\lambda_s=\lambda\binom{v-s}{t-s}\Big/\binom{k-s}{t-s}$, and in particular, $|\mathfrak{B}|=\lambda_0=\lambda\binom{v}{t}\Big/\binom{k}{t}$. Two t-designs with the same parame-