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Simple graded Lie algebras of finite depth

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Introduction

In this paper we classify the infinite dimensional simple graded Lie algebras of finite depth over an algebraically closed field K of characteristic zero.

A graded Lie algebra (GLA) $\mathfrak{g}=\bigoplus_{p\in Z}\mathfrak{g}_p$ is a Lie algebra $\mathfrak{g}$ endowed with a gradation $\{\mathfrak{g}_p\}_{p\in Z}$ such that dim $\mathfrak{g}_p<\infty$ and $[\mathfrak{g}_p, \mathfrak{g}_q]\subset\mathfrak{g}_{p+q}$. It is called simple if the underlying Lie algebra $\mathfrak{g}$ is simple. We say a GLA $\mathfrak{g}=\bigoplus_{p\in Z}\mathfrak{g}_p$ is of finite depth if the negative part $\mathfrak{g}_-=\bigoplus_{p<0}\mathfrak{g}_p$ is finite dimensional. Note that a GLA of finite depth having at least dimension two and no proper graded ideal is simple (see § 1). Note also that a simple GLA is necessarily transitive (a GLA $\mathfrak{g}=\bigoplus_{p\in Z}\mathfrak{g}_p$ is called transitive if for $x\in\mathfrak{g}_p$ $(p\geq 0)$, $[x, \mathfrak{g}_-]=0$ implies $x=0$).

Let $\mathfrak{g}=\bigoplus_{p\in Z}\mathfrak{g}_p$ be a simple GLA of finite depth. According to Cartan's classification of the simple infinite transitive pseudogroups, or rather according to its algebraic version, i. e., classification of the primitive infinite Lie algebras, completed by many authors (in particular, Singer-Sternberg [SS65], Kobayashi-Nagano [KN66], Guillemin-Quillen-Sternberg [GQS66], Morimoto-Tanaka [MT70]), we see that the underlying Lie algebra $\mathfrak{g}$ is isomorphic to one of the following series of simple Lie algebras:

1) $W(m)$: the Lie algebra of all the polynomial vector fields $\sum_{i=1}^{m} P_i\partial/\partial x_i$ with $P_i\in K[x_i,\dots,x_m]$.

2) $S(m)$: the subalgebra of $W(m)$ consisting of the vector fields which preserve the differential form $dx_1\wedge\dots\wedge dx_m$ $(m\geq 2)$;

3) $H(n)$: the subalgebra of $W(m)$ consisting of vector fields which preserve the differential form $\sum_{i=1}^{n} dx_i\wedge dx_{n+i}$, $m=2n$;

4) $K(n)$: the subalgebra of $W(m)$ consisting of vector fields which preserve the differential form $dx_m-\sum_{i=1}^{n} x_{i+n}dx_i$ $(m=2n+1)$ up to the multicative factors in $K[x_1,\dots,x_m]$.