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Obstruction to circle group actions preserving symplectic structure

Dedicated to Professor Haruo Suzuki on his sixtieth birthday

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1. Introduction.

In a previous paper [O], we discussed non-existence of compact connected Lie group actions on some syplectic manifolds from a point of view of a symplectic analogue of Kähler geometry. For example, closed sympletic manifolds with "negative first Chern class" admit no compact connected Lie group actions preserving symplectic structure. In the proof we used moment maps and lifting of group actions on certain complex line bundles.

In this note, we give another sufficient condition for non existence compact Lie group actions by using generalised moment map [MD]. Our result is the following

THEOREM. *Let (M, ω) be a closed symplectic manifold.*
1) *If the second homotopy group $\pi_2(M)$ vanishes, there is no circle group action on M preserving ω with non empty fixed point set*

$$M^{S^1}=\{p\in M | t\cdot p=p \text{ for any } t\in S^1\}.$$

2) *Moreover if any abelian subgroup of the fundamental group $\pi_1(M)$ is cyclic, there is no circle group action on M preserving ω. Therefore there is no compact connected Lie group action preserving ω.*

These conditions are satisfied for closed negatively curved manifolds and it is known that they admit no circle group actions. The author is very grateful to Professor Akio Hattori for advice and encouragement. He also would like to thank Professor Dusa McDuff for sending him a preprint with helpful advice.