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An example of a regular Cantor set whose difference set is a Cantor set with positive measure

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Dedicated to Professor Haruo Suzuki on his 60th birthday

§ 0. Introduction

In this paper, we give an example of a regular Cantor set whose self-difference set is a Cantor set and, at the same time, has a positive measure. This is a counter example of one of the questions posed by J. Palis related to homoclinic bifurcation of surface diffeomorphisms.

In [PT], Palis-Takens investigated homoclinic bifurcation in the following context. Let M be a closed 2-dimensional manifold. We say a C^r-diffeomorphism $\phi: M\rightarrow M$ is *persistently hyperbolic* if there is a C^r-neighborhood $\mathscr{U}$ of ϕ such that for every $\psi\in\mathscr{U}$, the non-wandering set $\Omega(\psi)$ is a hyperbolic set (refer [PM] for the definitions and the notations of the terminologies of dynamical systems). Let $\{\phi_\mu\}_{\mu\in\mathbf{R}}$ be a 1-parameter family of C^2-diffeomorphisms on M. We say $\{\phi_\mu\}_{\mu\in\mathbf{R}}$ has a *homoclinic* Ω*-explosion* at $\mu=0$ if:

(i) For $\mu<0$, ϕ_μ is persistently hyperbolic;

(ii) For $\mu=0$, the non-wandering set $\Omega(\phi_0)$ consists of a (closed) hyperbolic set $\tilde{\Omega}(\phi_0)=\lim_{\mu\uparrow 0}\Omega(\phi_\mu)$ together with a homoclinic orbit of tangency $\mathscr{O}$ associated with a fixed saddle point p, so that $\Omega(\phi_0)=\tilde{\Omega}(\phi_0)\cup\mathscr{O}$; the product of the eigenvalues of $d\phi_0$ at p is different from one in norm;

(iii) The separatrices have quadratic tangency along $\mathscr{O}$ unfolding generically; $\mathscr{O}$ is the only orbit of tangency between stable and unstable separatrices of periodic orbits of ϕ_0.

Let Λ be a basic set of a diffeomorphism ψ on M. $d^s(\Lambda)$ ($d^u(\Lambda)$) denotes the Hausdorff dimension in the transversal direction of the stable (unstable) foliation of the stable (unstable) manifold of Λ (refer [PM] for the precise definition), and is called the stable (unstable) *limit capacity*. Let B denote the set of values $\mu>0$ for which ϕ_μ is not persistently hyperbolic.