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Smooth $SL(n, C)$ actions on $(2n-1)$-manifolds

Dedicated to Professor Haruo Suzuki on his 60th birthday

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0. Introduction.

Smooth $SL(2, C)$ actions on closed connected 3-manifolds are classified by T. ASOH [1].

In this paper, we shall classify smooth $SL(n, C)$ actions on closed connected $(2n-1)$-manifolds for $n\geq 3$. We shall show that such a manifold is equivariantly diffeomorphic to the lens space $L^{2n-1}(p)$ or the product space $P_{n-1}(C)\times S^{1}$, with certain $SL(n, C)$ action. Our main result is stated in Theorem 3.

1. Certain subgroups of $SU(n)$.

Let K be a closed connected proper subgroup of $SU(n)$, and suppose $\dim SU(n)/K\leq 2n-1$, that is, $\dim K\geq n(n-2)$. Notice that the inclusion $i:K\rightarrow SU(n)$ gives a unitary representation of K.

Suppose first that the representation i is reducible, that is, there is a positive integer k such that $2k\leq n$ and K is contained in $S(U(k)\times U(n-k))$ up to an inner automorphism of $SU(n)$. If $k\geq 2$, then

$$\begin{aligned}2n-1<kn\leq 2k(n-k)&=\dim SU(n)/S(U(k)\times U(n-k))\\&\leq\dim SU(n)/K.\end{aligned}$$

Hence we obtain $k=1$. Moreover, we see that K coincides with $SU(n-1)$ or $S(U(1)\times U(n-1))$ up to an inner automorphism of $SU(n)$, by the fact that there is no closed subgroup of codimension 1 in $SU(n-1)$ for each $n\geq 3$.

Next we consider the case that the representation i is irreducible. We see that K is semi-simple, because K is contained in $SU(n)$.

Suppose that K is not simple. Then, there are closed normal subgroups H_{1}, H_{2} of K and irreducible unitary representations $r_{j}:H_{j}\rightarrow U(n_{j})$ such that the tensor product $r_{1}\otimes r_{2}$ is equivalent to $i\pi$, where $n=n_{1}n_{2}$, $n_{j}\geq 2$ and $\pi:H_{1}\times H_{2}\rightarrow K$ is a covering projection.

By our assupmtion, we obtain