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# Expansive foliations

Dedicated to Professor Haruo Suzuki on his sixtieth birthday

Takashi INABA and Nobuo TSUCHIYA
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## Introduction

Expansiveness of homeomorphisms and flows has been studied by various authors in the field of dynamical systems. In this note we introduce this notion into the foliation theory and examine its influence on the topology of leaves. In § 1 we give a precise definition of expansive foliations. In § 2 we restrict our attention to the case of codimension one foliations and show that in this case topological structures of foliations completely characterize the expansiveness. As corollaries of this result we obtain that the geometric entropy ([GLW]) of a codimension one expansive foliation is positive and that the fundamental group of a manifold admitting a codimension one expansive foliation has exponential growth. In § 3 we define another notion called strong expansiveness. We show that for strongly expansive foliations results similar to (although somewhat weaker than) those obtained in § 2 hold in all codimensions.

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## 1. Definition

First we treat the case of codimension one. Let $M$ be a closed $C^\infty$ Riemannian manifold and $\mathscr{F}$ a codimension one $C^r$, $r \geq 0$, foliation on $M$. Fix a one dimensional foliation $\mathscr{T}$ transverse to $\mathscr{F}$. Throughout this note we assume that all leaves of $\mathscr{F}$ and $\mathscr{T}$ are of class $C^1$. A curve (resp. an embedded curve) contained in a leaf of $\mathscr{F}$ (resp. $\mathscr{T}$) is called an *$\mathscr{F}$-curve* (resp. a *$\mathscr{T}$-arc*). A continuous map $F:[0,1]\times[0,1]\to M$ is called a *fence* if $F|[0,1]\times\{t\}$ is a $C^1$ $\mathscr{F}$-curve for all $t\in[0,1]$ and $F|\{s\}\times[0,1]$ is a $\mathscr{T}$-arc for all $s\in[0,1]$. $F|[0,1]\times\{t\}$ is called a *horizontal curve* (the *lower side* if $t=0$, the *upper side* if $t=1$) of $F$ and $F|\{s\}\times[0,1]$ is called a *vertical arc* (the *left side* if $s=0$, the *right side* if $s=1$) of $F$. The lower, upper, left or right side of $F$ is denoted by $l(F)$, $u(F)$, $\lambda(F)$