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Twisted linear actions on complex Grassmannians

Dedicated to Professor Haruo Suzuki on his 60th birthday

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0. Introduction

In this paper, we shall study twisted linear actions of noncompact Lie groups on complex Grassmannians as the sequel to [3]. The first example of twisted linear actions on spheres was given by F. Uchida (cf. [5], [6]) and later the author (cf. [3]) gave such an example over complex (or quaternionic) projective spaces. It seems interesting to examine twisted linear actions on simply connected compact irreducible symmetric spaces of rank greater than one as well. The paper is organized as follows; some preliminary facts are collected to describe complex Grassmannians for our use in Section 1, the twisted linear actions are dealt with in Section 2 and 3.

One of the main results is that any twisted linear actions of compact Lie groups on complex Grassmannians are equivalent to the linear actions (cf. Theorem 2.2). On the contrary we emphasize that, as well as on the complex projective spaces, there are uncountably many topologically inequivalent twisted linear C^{ω}-actions of the noncompact Lie group $SL(n, \boldsymbol{C})$ on the complex Grassmannian $\boldsymbol{G}_{nk,m}$ of all m-dimensional linear subspaces in the nk-dimensional complex Enclidean space $\boldsymbol{C}^{nk}$, where $n>mk$ and $k>1$ (cf. Theorem 3.3). For complex Grassmannians, the author could not obtain the results corresponding to Theorem 3.3 and 3.5 of [3]. For quaternionic Grassmannians, our methods can not be used, since the quaternion field is noncommutative. The author does not know how twisted linear actions of Lie groups on quaternionic Grassmannians are defined.

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1. A description of complex Grassmannian

1.1. Let $M(n, m; \boldsymbol{C})$ be the set of all complex matrices of type $n\times m$ and put $M_n(\boldsymbol{C})=M(n, n; \boldsymbol{C})$. For $X, Y\in M(n, m; \boldsymbol{C})$, we define their hermitian inner product by $\langle X, Y\rangle=\mathrm{trace}(X^{*}Y)$ and the norm of X by