Hokkaido Mathematical Journal Vol. 21(1992) p. 187-194

Weakly complex manifolds with semi-free S^1-action whose fixed point set has complex codimension 2

Dedicated to Professor Haruo Suzuki on his sixtieth birthday

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(Received April 19, 1991, Revised September 11, 1991)

1. Introduction

A weakly complex manifold means a smooth manifold whose tangent bundle is stably equivalent to a complex vector bundle. Let M^{2n} be a $2n$-dimensional closed weakly complex manifold and let $\varphi: S^1\times M^{2n}\to M^{2n}$ be a smooth semi-free S^1-action which preserves the complex structure. We denote this manifold by the pair (M^{2n}, φ). Let $F(M^{2n}, \varphi)=F_1\cup F_2\cup\cdots\cup F_s$, where $F_i(i=1,2,\cdots,s)$ is a fixed point set component. Each F_i has an S^1-invariant weakly complex structure. Then we have the following theorem by the Kamata's formula [2].

THEOREM 1. *Let k be a positive integer and let (M^{2n}, φ) be a weakly complex semi-free S^1-manifold. Let $dim_C F_i=n-2k$ $(i=1, \cdots,s)$. Then the Chern number $c_1^n[M^{2n}]\equiv 0$ mod $(2k)^{2k}$.*

Next in this paper we study, up to mod 2 bordism, those manifolds with semi-free S^1-action with the property that all the components of the fixed point set have the same complex codimension 2.

Let $\mathscr{U}_*$ be the bordism ring of closed weakly complex smooth manifolds. It is known that the bordism ring $\mathscr{U}_*$ is generated by a set of bordism classes $\{[CP(k)], [H_{m,n}(C)]; k\geq 1, n\geq m>1\}$, where $CP(k)$ is the k dimensional complex projective space and $H_{m,n}(C)$ is the Milnor hypersurface in $CP(m)\times CP(n)$. For our purpose, we calculate a base of the mod 2 weakly complex bordism ring $\mathscr{U}_*\otimes Z_2$. Let $(n_1,n_2,\cdots,n_k)$ be a k-tuple of non negative integers. We denote by $CP(n_1,n_2,\cdots,n_k)$ the complex projective space bundle $CP(\lambda_1\oplus\lambda_2\oplus\cdots\oplus\lambda_k)$ associated to the bundle $\lambda_1\oplus\lambda_2\oplus\cdots\oplus\lambda_k$ over $CP(n_1)\times CP(n_2)\times\cdots\times CP(n_k)$, where $\lambda_i(i=1,2,\cdots,k)$ is the pullback of the canonical line bundle over the ith factor.

Now we define an ideal $\mathscr{T}$ in $\mathscr{U}_*\otimes \boldsymbol{Z}_2$ as follows.