Hokkaido Mathematical Journal Vol. 21 (1992) p. 103-116

KO-theory of Hermitian symmetric spaces

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(Received January 21, 1991)

Dedicated to Professor Haruo Suzuki on his 60th birthday

§ 1. Introduction

Our purpose of this paper is the determination of KO-theory of the compact irreducible Hermitian symmetric spaces. The spaces are classified by E. Cartan as follows:

$AIII$	$M_{m,n}=U(m+n)/(U(m)\times U(n))$	
BDI	$Q_n \;=SO(n+2)/(SO(n)\times SO(2))$	$(n\geq 3)$
CI	$Sp(n)/U(n)$	$(n\geq 3)$
$DIII$	$SO(2n)/U(n)$	$(n\geq 4)$
$EIII$	$=E_6/(Spin(10)\cdot T^1)$	$(Spin(10)\cap T^1\cong \boldsymbol{Z}_4)$
$EVII$	$=E_7/(E_6\cdot T_1)$	$(E_6\cap T^1\cong \boldsymbol{Z}_3)$.

Bott showed their cohomology rings have no torsion and no odd dimensional part. The integral cohomology rings are determined by [2], [9] and [10], while the actions of the squaring operations on them are determined in [5]. In [6], we compute the KO-theory of $M_{m,n}$. Here we show:

THEOREM 1. *Let X be a compact irreducible Hermitian symmetric space, then its Atiyah-Hirzebruch spectral sequence for $KO^*(X)$:*

$$E_r^{*,*}(X)\Rightarrow KO^*(X)$$

has nontrivial differential d_r only for $r=2$.

Let $H^*(X)$ be the modulo 2 cohomology ring of X. When the odd dimensional parts of $H^*(X)$ are trivial, $Sq^2Sq^2(=Sq^3Sq^1)$ vanishes on $H^*(X)$, and $(H^*(X), Sq^2)$ is a differential module. For the proof of Theorem 1 we compute the (co)homology group $H(H^*(X); Sq^2)$, which is isomorphic to $E_3^{*,-1}(X)$, and show the differentials d_r $(r\geq 3)$ are trivial for each X.

By Theorem 1, KO^* (X) is obtained from $E_3^{*,*}(X)$. Consequently the groups $H^*(X)$ and $H(H^*(X), Sq^2)$ determine $KO^*(X)$ in the following corollary.