Hokkaido Mathematical Journal Vol. 22 (1993) p. 199-209

A new algorithm derived from the view-point of the fluctuation-dissipation principle in the theory of KM_2O-Langevin equations

Yasunori OKABE
(Received October 16, 1992)

§1. Introduction and statements of results

We have constructed in [4] a theory of KM_2O-Langevin equations for multi-dimensional weakly stationary processes with discrete time, and from the view-point of the so-called fluctuation-dissipation theorem in irreversible statistical physics ([2]), we have established *a fluctuation-dissipation theorem* which gives a relation between the fluctuant and deter-ministic terms in the KM_2O-Langevin equation. Such *a fluctuation-dissipation theorem* had already been found as *the Levinson-Whittle-Wiggins-Robinson algorithm* for *the fitting problem of AR-models* in the field of system, control and information ([3], [1], [10], [11]). Sublimating a certain philosophical structure behind our fluctuation-dissipation theorem to form *the fluctuation-dissipation principle*, we have applied the theory of KM_2O-Langevin equations to data analysis and developped *a stationary analysis* as well as *a causal analysis* ([7], [6]). Furthermore, on these lines, we have solved the non-linear prediction problem for one-dimensional strictly stationary processes with discrete time and developped *a prediction analysis* as our third project in data analysis ([5], [9], [8]).

Let $\mathbf{X}=(X(n); n\in\mathbf{Z})$ be an $\mathbf{R}^d$-valued weakly stationary process on a probability space $(\Omega, \mathscr{B}, P)$ with expectation vector zero and covariance matrix function R:

$$(1.1) \qquad R(m-n)\equiv E(X(m)\,{}^tX(n)) \qquad (m, n\in\mathbf{Z}),$$

where d is any fixed natural number.

For each $n\in\mathbf{N}$, a block Toeplitz matrix $S_n\in M(nd; \mathbf{R})$ is defined by

This research was partially supported by Grant-in-Aid for Science Research No. 03452011 and No. 04352003, the Ministry of Education, Science and Culture, Japan.