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The deficiencies of algebroid functions of order less than one

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Abstract. T. Sato [4] generalized Theorem A to algebroid function under some additional condition. In this paper, we give an extension of Theorem A without any additional condition which improves Sato's results and we also generalized Theorem B.

(I) Introduction

We assume that the reader is familiar with the fundamental concepts of Nevanlinna's theory of meromorphic functions and in particular with the most usual of its symbols:

$$\log^+;\ m(r, w);\ n(r, w);\ N(r, w);\ T(r, w).$$

We shall use, whenever this is possible without ambiguity, the simplified notations $m(r, a)$; $n(r, a)$; $N(r, a)$; $T(r)$ in place of $m(r, 1/(w-a))$; $n(r, 1/(w-a))$; $N(r, 1/(w-a))$; $T(r, w)$.

The letters ρ and μ denotes the order and lower order of $w(z)$, respectively:

$$\rho=\overline{\lim_{r\to\infty}}\frac{\log T(r, w)}{\log r};\quad \mu=\underline{\lim_{r\to\infty}}\frac{\log T(r, w)}{\log r}.$$

The Nevanlinna deficiency $\delta(a, w)$ of the value a for the function $w(z)$ is, by definition,

$$\delta(a, w)=1-\overline{\lim_{r\to\infty}}\frac{N(r, a)}{T(r, w)}.$$

The Valiron deficiency $\Delta(a, w)$ of a is,

$$\Delta(a, w)=1-\underline{\lim_{r\to\infty}}\frac{N(r, a)}{T(r, w)}.$$

A value a for which $\Delta(a, w)>0$ is said to be deficient in the sense of Valiron; if $\delta(a, w)>0$, then the value a is deficient in Nevanlinna's sense.

Edrei and Fuchs proved [1; Theorem 1, P. 233].

THEOREM A. *Let $w(z)$ be a meromorphic function of order ρ satis-*