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A decomposition theorem of operators for variegations

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1. Introduction

We consider a decomposition theorem of (bounded linear) operators on Hilbert spaces by analogy with variety in unversal algebra. It is known that any contraction is decomposed into the direct sum of the unitary part and the completely non-unitary part (B. Sz.-Nazy and C. Foias [9]). A general theory of decompositions of operators are developed by J. Ernest [4], A. Brown, C.-K. Fong and D. W. Hadwin [2], [6]. W. Szymanski [10] also studied the canonical decomposition of operator-valued functions in Hilbert spaces. Following his ideas, M. Fujii, M. Kajiwara, Y. Kato, F. Kubo and S. Maeda considered decompositions of operators ([5], [7], [8]). In this paper we shall give another condition on classes of operators to have the canonical decomposition. Finally we should remark that a recent work [1] by J. Agler is very interesting and has a relation with our paper.

2. Decomposable function

We consider a property $\mathscr{S}$ on operators and identify it with the class of all operators having the property $\mathscr{S}$. Many properties are defined by equations of non-commutative polynomials. A property $\mathscr{S}$ is called *algebraically difinite* if there is a family G of non-commutative polynomials $p(x, y)$ such that $T\in\mathscr{S}\Leftrightarrow p(T, T^*)=0$ for all $p\in G$, cf. [5] and [8]. More generally we consider a class of operators involving more general "functions" called decomposable functions invented by A. Brown, C. K. Fong and D. W. Hadwin [2], [6]. Let $B(K)$ be the set of all bounded linear operators on a Hilbert space K.

DEFINITION ([2]). Let H be a separable, infinite dimensional Hilbert space. A *decomposable function* on H is a function ϕ on $\cup\{B(M)\,; M$ is a subspace of $H\}$ such that

(a) $\phi(B(M))\subset B(M)$ for every subspace M of H,

(b) if $T\in B(H)$ and M is a reducing subspace of T, then M reduces $\phi(T)$ and $\phi(T|_M)=\phi(T)|_M$