Hokkaido Mathematical Journal Vol. 24 (1995) p. 575–615

The asymptotic behaviour of the radially symmetric solutions to quasilinear wave equations in two space dimensions

(Dedicated to Professor Kôji Kubota on his 60th birthday)

Akira HOSHIGA[1]

(Received January 31, 1995; Revised April 12, 1995)

Abstract. In this paper, we study the behaviour of solutions to quasilinear wave equations in two space dimensions. We obtain blow-up results near the wave front. More precisely, any radially symmetric solution with small initial data is shown to develop singularities in the second order derivatives in finite time, while the first order derivatives and itself remain small. Moreover, we succeed to represent the solution explicitely near the blowing up point.

Key words: Quasilinear wave equation.

1. Introduction

This paper deals with the initial value problem:

$$u_{tt} - c^2(u_t, u_r)\left(u_{rr} + \frac{1}{r}u_r\right) = \frac{1}{r}u_r G(u_t, u_r),$$
$$(r,t) \in (0,\infty) \times (0, T_\varepsilon), \qquad (1.1)$$

$$u(r,0) = \varepsilon f(r), \qquad u_t(r,0) = \varepsilon g(r), \qquad r \in (0,\infty) \qquad (1.2)$$

where

$$c^2(u_t, u_r) = 1 + a_1 u_t^2 + a_2 u_t u_r + a_3 u_r^2 + O(|u_t|^3 + |u_r|^3),$$
$$G(u_t, u_r) = O(|u_t|^2 + |u_r|^2)$$

near $u_t = u_r = 0$ and the initial data are smooth and have compact support. The equation (1.1) is the radially symmetric form of quasi-linear wave equations in two space dimensions. In [2], we have shown that the smooth solution to the initial value problem (1.1) and (1.2) exists almost globally,

[1]Partially supported by the Suhara Memorial Foundation

1991 Mathematics Subject Classification : 35A05, 35A07, 35B45, 35L05, 35L15.