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2-Type flat integral submanifolds in $S^7(1)$

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Abstract. This paper determines all flat, mass-symmetric, 3-dimensional 2-type submanifolds of the unit sphere $S^7(1)$ which are integral submaniifolds of the canonical contact structure.

Key words: integral submanifolds, finite type submanifolds.

1. Introduction

In [5,6] Bang-Yen Chen introduced the notion of submanifolds of finite type. Let M be a submanifold of Euclidean space E^n and Δ the Laplacian of the induced metric. M is said to be of *finite type* if its position vector field x has a decomposition of the form

$$x = x_0 + x_1 + \cdots + x_k$$

where x_0 is a constant vector and $\Delta x_i = \lambda_i x_i$. Assuming the λ_i to be distinct we say that M is of *k-type*.

The theory of finite type submanifolds has become an area of active research. The first results on this subject have been collected in the book [6]; for a recent survey, see [7]. In particular, there is the problem of classification of low type submanifolds which lie in a hypersphere. Far from being solved in general, there are many partial results which contribute to the solution of this problem. For instance, by the well-known result of Takahashi [10], 1-type submanifolds are characterized as being minimal in a sphere.

However, classification of even 2-type spherical submanifolds seems to be virtually impossible. A compact submanifold M^n of a hypersphere S^m of E^{m+1} is said to be *mass-symmetric* if the center of mass of M^n in E^{m+1} is the center of S^m in E^{m+1}. Note that the only 2-type surface in S^3 is the flat torus $S^1(a) \times S^1(b)$, $a \neq b$, while a 2-type mass-symmetric integral surface in S^5 is locally the product of a circle and a helix of order 4, or

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