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# Existence and asymptotic behavior of weak solutions to strongly damped semilinear hyperbolic systems

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**Abstract.** Weak solutions to a strongly damped semilinear hyperbolic system are constructed by the method of semi-descretization in time variable combining with variational calculus. The asymptotic behavior of solutions is also investigated and the decay property under the homogeneous boundary condition is shown by the discrete energy method.

*Key words*: semilinear hyperbolic systems, a strongly damping term, asymptotic behavior

## 1. Introduction

Let $\Omega$ be a bounded domain of $\mathbf{R}^k$ with Lipschitz boundary $\partial\Omega$. We consider the following system of hyperbolic equations for a map $u:\Omega\times(0,\infty)\rightarrow\mathbf{R}^{\ell}$:

$$\begin{aligned} a_{ij}(x)D_t^2u^i(x,t) &- D_\beta\left(b_{ij}^{\alpha\beta}(x)D_\alpha u^i(x,t)\right) \\ &+ c_{ij}(x)\|u(x,t)\|_c^{m-2}u^i(x,t) - D_tD_\beta(f_{ij}^{\alpha\beta}(x)D_\alpha u^i(x,t)) = 0 \\ &\text{in } \Omega\times(0,\infty),\ j=1,\ \cdots,\ \ell, \end{aligned} \tag{1.1}$$

where $D_t=\partial/\partial t$, $D_\alpha=\partial/\partial x^\alpha$, $\|u(x,t)\|_c=\left(c_{ij}(x)u^i(x,t)u^j(x,t)\right)^{1/2}$ and $m>1$. Here and in the following, summation over repeated indices is understood, the greek indices run from 1 to $k$, and the latin ones from 1 to $\ell$. We assume that the coefficients $a_{ij}(x)$, $b_{ij}^{\alpha\beta}(x)$, $c_{ij}(x)$ and $f_{ij}^{\alpha\beta}$ are bounded functions defined on $\Omega$ and satisfy the conditions

$$\left\{\begin{aligned} a_{ij}(x)\xi^i\xi^j &\geq \lambda_0|\xi|^2 && \text{for all } \xi\in\mathbf{R}^{\ell}, \\ b_{ij}^{\alpha\beta}(x)\eta_\alpha^i\eta_\beta^j &\geq \lambda_1|\eta|^2 && \text{for all } \eta\in\mathbf{R}^{k\ell}, \\ c_{ij}(x)\xi^i\xi^j &\geq \lambda_2|\xi|^2 && \text{for all } \xi\in\mathbf{R}^{\ell}, \\ f_{ij}^{\alpha\beta}(x)\eta_\alpha^i\eta_\beta^j &\geq \lambda_3|\eta|^2 && \text{for all } \eta\in\mathbf{R}^{k\ell}, \end{aligned}\right. \tag{1.2}$$

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