

## Singular limits in the data space for the equations of magneto-fluid dynamics

Bruno RUBINO

(Received June 29, 1994; Revised November 22, 1994)

**Abstract.** We discuss the singular limit of the incompressible magneto-fluid periodic motion with respect to the Alfvén number. Viscous and inviscid magneto-fluids are treated indistinctly. We determine the limiting system under the natural assumptions on the initial data. Finally, we apply the theory of Beirão da Veiga [7, 9] and prove convergence in the data space.

*Key words:* Alfvén number, incompressible viscous and inviscid magneto-fluid, kinematic and magnetic Reynolds' number, singular limit.

### 1. Introduction

In this paper we study the behavior of the solution  $(v, H)$  for the following equations of motion of an incompressible viscous magneto-fluid in the  $n$ -dimensional torus,  $n \geq 2$ , as the Alfvén number tends to zero:

$$\begin{cases} \partial_t v + (v, \nabla)v + \nabla p + \alpha^2 H \times \operatorname{curl} H - \sigma \Delta v = 0 \\ \partial_t H + (v, \nabla)H - (H, \nabla)v - \mu \Delta H = 0 \\ \operatorname{div} v = 0, \quad \operatorname{div} H = 0 \\ v(0) = v_0, \quad H(0) = H_0 \end{cases} \quad (1.1)$$

where  $v = v(t, x)$  is the fluid velocity,  $H = H(t, x)$  the magnetic field,  $p = p(t, x)$  the pressure and the parameters  $\alpha$ ,  $\sigma$  and  $\mu$  are respectively the reciprocal of the Alfvén number, of the kinematic Reynolds' number and of the magnetic Reynolds' number.

We assume that  $\alpha \geq 1$ ,  $\sigma \in [0, \sigma_0]$ ,  $\mu \in [0, \mu_0]$  for arbitrarily fixed constants  $\sigma_0, \mu_0$ . Since the viscosity coefficients  $\sigma$  and  $\mu$  can assume any value in the above ranges, we study simultaneously the viscous and the non viscous magneto-fluids.

We are interested in studying the limits of  $(v, H)$  as  $\alpha \rightarrow \infty$  and the viscosity parameters  $\sigma$  and  $\mu$  converge to  $\bar{\sigma} \geq 0$ ,  $\bar{\mu} \geq 0$ .