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Solutions of the fifth Painlevé equation I[1]

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Abstract. Here we determine all the transcendental classical solutions of the fifth Painlevé equation.

Key words: Painlevé equations, classical solutions, the condition (J).

Introduction

In our previous paper [21] (see also [18], [19]), we emphasized the importance of the determination of all the classical solutions of the Painlevé equations in connection with the proof of their irreducibility in the sense of Painlevé (cf. [17]). In this paper I and the next paper II [23], following our previous papers [21], [22] on the solutions of the second, third and fourth Painlevé equations, we determine all the classical solutions of the fifth Painlevé equation. The determination of the classical solutions consists of that of the algebraic solutions and that of the transcendental classical solutions. In the paper II we discuss the former; in this paper I we discuss the latter. In these papers we follow the terminology of [21].

The fifth Painlevé equation $P_V(\alpha,\beta,\gamma,\delta)$ is given by

$$\begin{aligned}\frac{d^2Q}{dt^2} &= \left(\frac{1}{2Q}+\frac{1}{Q-1}\right)\left(\frac{dQ}{dt}\right)^2-\frac{1}{t}\frac{dQ}{dt}\\ &\quad+\frac{(Q-1)^2}{t^2}\left(\alpha Q+\frac{\beta}{Q}\right)+\frac{\gamma}{t}Q+\delta\frac{Q(Q+1)}{Q-1},\end{aligned}$$

where $\alpha,\beta,\gamma,\delta$ denote complex numbers. It is known ([3], [12]) that the equation $P_V(\alpha,\beta,\gamma,0)$ is reduced to the third Painlevé equation, so that we may assume $\delta=-\frac{1}{2}$ without loss of generality (see [6], [13]). The equation $P_V(\alpha,\beta,\gamma,-\frac{1}{2})$ is equivalent to a system $\tilde{S}(\mathbf{v})$ of ordinary differential

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