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Nonhomogeneity of Picard dimensions on the half ball

Hideo IMAI

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We denote by H^m the upper half space $\{x=(x_1, \cdots, x_m)\in R^m : x_m>0\}$ in the Euclidean m-space $R^m(m\geq 2)$ and by $\hat{H}^m$ the closure of H^m with respect to the one point compactification of R^m. Setting $\delta H^m=\hat{H}^m\backslash H^m$, we may view $\{x\in\hat{H}^m : x_m=0\}$ as a subset of an ideal boundary δH^m of H^m and the origin $x=0$ as an ideal boundary point of H^m. Take the upper half ball $U_s^+=\{x=(x_1, \cdots, x_m)\in H^m : |x|<s\}$ $(0<s\leq 1)$ which may be regarded as a relative neighbourhood of the ideal boundary point $x=0$ of H^m. The set $\Gamma_s^+\equiv\{x\in H^m : |x|=s\}$ is a relative boundary of U_s^+ and $\gamma_s^+\equiv\{x\in\delta H^m : |x|\leq s\}$ is an ideal boundary of U_s^+. Therefore the boundary ∂U_s^+ of U_s^+ and the closure $\bar{U}_s^+$ of U_s^+ in $\hat{H}^m$ are $\Gamma_s^+\cup\gamma_s^+$ and $U_s^+\cup\Gamma_s^+\cup\gamma_s^+$, respectively. In particular we set $U_1^+=U^+$ and $\Gamma_1^+=\Gamma^+$. By a *density* $P(x)$ on U_s^+ we mean a locally Hölder continuous function $P(x)$ defined on $\bar{U}_s^+\backslash\{0\}$. Hence P may have a singularity at the ideal boundary point $x=0$.

Consider the time independent Schrödinger equation

$$L_P u(x) \equiv -\triangle u(x)+P(x)u(x) = 0 \tag{1}$$

defined on $\bar{U}_s^+\backslash\{0\}$, where $\triangle$ is the Laplacian $\triangle=\sum_{i=1}^m \partial^2/\partial x_i^2$. We are interested in the class $PP(U_s^+)$ of nonnegative solutions of (1) in U_s^+ with vanishing boundary values on $\partial U_s^+\backslash\{0\}$. The first P indicates the dependence of the class on the density P and the second P stands for the initial of the term positive (nonnegative) so that the class associated with another density Q is denoted by $QP(U_s^+)$. It is convenient to consider the subclass $PP_1(U_s^+)\equiv\{u\in PP(U_s^+) : u(x_s)=1\}$, where x_s is an arbitrary point fixed in U_s^+. Since $PP_1(U_s^+)$ is a compact and convex set with respect to almost uniform convergence on U_s^+, we can consider the set $ex. PP_1(U_s^+)$ of extreme points of $PP_1(U_s^+)$ and the cardinal number $\#(ex. PP_1(U_s^+))$ of $ex. PP_1(U_s^+)$ which will be referred to as the *Picard dimension* of (U_s^+, P) at $x=0$, $\dim(U_s^+, P)$ in notation:

$$\dim(U_s^+, P) = \#(ex. PP_1(U_s^+)).$$

In particular we say that the *Picard principle* is valid for (U_s^+, P) at $x=0$