A class of singular integral operators
with rough kernel on product domains*

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Abstract. The $L^2$-boundedness for a class of singular integral operators with rough kernel on product domain is discussed in terms of block decompositions. The main result is an improvement of corresponding one on $L^2$-boundedness due to J. Duoandikoetxea.

1. Introduction

It is well known that the singular integral operators on product domain $R^n \times R^m$ defined by

$$Tf(x, y) = p.v. \int_{R^n \times R^m} K(\xi, \eta)f(x-\xi, y-\eta)d\xi d\eta$$

are bounded on $L^p(R^n \times R^m)$, $1 < p < \infty$, provided

$$K(x, y) = \Omega(x/|x|, y/|y|)|x|^{-n}|y|^{-m},$$

$\Omega$ is homogeneous of degree zero, $\int_{S^{n-1}} \Omega(u, v)du = \int_{S^{m-1}} \Omega(u, v)dv = 0$, and some regularity conditions on $\Omega$ are assumed (see [2]). The $L^p$-boundedness of $T$ with the rough condition $\Omega \in L^q(S^{n-1} \times S^{m-1})$ instead of regularity is obtained in [1]. In this paper, we shall use the method of block decomposition for functions to improve the result of $L^2$-boundedness above. It should be pointed out that the method of block decomposition for functions is originated by M. H. Taibleson and G. Weiss in the study of the convergence of the Fourier series (see [6]). Latter on, many applications of the block decomposition to Harmonic analysis were discovered (see [5]). For example, a sort of method related to block decompositions is applied to study the $L^p$-boundedness of singular integral operators with rough kernel in [3]-[4]. Thus, this paper can also be regarded as generalization of the one-parameter results in [3]-[4].

Let us begin with the definition of $q$-block on $S^{n-1} \times S^{m-1}$.

Definition 1 A function $b(u, v)$ on $S^{n-1} \times S^{m-1}$ is called a $q$-block,